Flow characterization by ultrasound scattering on transducer array:
a new tool in turbulence

Thesis submitted in partial fulfillment
of the degree of Doctor of Philosophy

by

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Submitted to the Feinberg Graduate School
of the Weizmann Institute of Science
Rehovot 2004
Abstract

This work presents a new approach to ultrasound measurements of either single vortex or turbulent flow structures. The tool is based on multi-channel acquisition of coherent sound signals from a transducer array of linear geometry. Since the acquisition is simultaneous to all acoustic channels it is possible to utilize the scheme of wave propagation known as the Fresnel-Huygens construction, and to obtain the waveform at infinity. Via such step in analysis of the scattering field of sound the structure functions of the flow are obtained in the Fourier space.

The sound detection system is occupied to characterize the flow between either co-rotating or counter rotating plates (the so called von Karman flows). In parallel, we study the flow using Particle Image Velocimetry (PIV) and Hot-Wire Anemometry (HWA) in order to compare results with the sound method and for gaining additional information. The sound detection tool is found a significant method in the study of turbulent flows in liquid.

The information collected about the turbulent flow includes the energy spectrum, its scaling property, and the fitted dissipation values compared with the values extracted from the third order structure function of velocity difference. The integral scale is extracted from the level of fluctuations of velocity and the sound phase shift measured in parallel. We check the extension of the Taylor hypothesis and apply it for interpretation of the moments of sound phase shift fluctuations. The spatio-temporal map of velocity difference is introduced. The measures of intermittency are also presented by showing the probability density functions and the extended self similarity exponents.
Acknowledgements

I would like to express my appreciation to Professor Victor Steinberg for his expert guidance and his vision of the scientific project. His insisting on achievements and sense of judgment influenced my work. I thank him for what he taught me and for trusting in me.

I enjoyed working in the group and meeting over the years with Arun Roy, Corinne Chevallard, Alexander Groisman, with Teodor Burghlea, who contributed his point of view and considerable knowledge over long discussions, with Yuri Burnishev, who was willing to solve any problem, with Sergey Gerashchenko, Vasili Kantsler, Sergiy Kapishnikov, Enrico Segre, and with Sergey Lukaschuk, who guided my first steps in acoustics. I benefited from everyone.

I admit a valuable contribution from Enrico Segre of his PIV software and for helping to set it on correct operation. I thank also Yuri Myasoedov for setting up with me an evaporation system that was once needed.

I am grateful for all the Weizmann members who helped and supported my requests. I learned from the advices of Yossi Choppen in the workshop. As a student I thank all the lecturers that passed me a shred of their knowledge.

Finally, I would like to express my profound gratitude to my wife Hilla for going with me in this way.
# Table of contents

1. **Introduction**  
   1

2. **Theory of ultrasound scattering by flow**  
   2.1. Sound scattering by a finite width beam  
   2.2. Alternative approach to find velocity and circulation: phase shift slope in a forward scattering.  
   2.3. Construction of a far field wave function from the near field measurements.  
   2.4. Conclusion  
   13

3. **Experimental set-up and measuring techniques**  
   3.1. Hydrodynamic set-up  
   3.2. Ultrasound probe  
     3.2.1. General scheme  
     3.2.2. Calibration  
   3.3. Particle Image Velocimetry (PIV)  
     3.3.1 Measurement scheme in single vortex mode  
     3.3.2 Measurement scheme in counter rotation mode  
     3.3.3. Calibration  
   3.4. Hot-Wire Anemometry (HWA)  
     3.4.1. General Scheme  
     3.4.2. Calibration  
   24

4. **Information on a single vortex extracted from sound scattering and compared with PIV**  
   4.1. Method of data analysis  
   4.2. Characterization of the flow by PIV  
   4.3. Velocity and circulation measurements via phase shift.  
   4.4. Sound scattering data.  
   4.5. Far field construction of sound scattering field and its relation to velocity and vorticity of the scattering region.  
   4.6. Dynamics of a vortex measured by sound scattering  
   38

5
4.7. Conclusions.

5. Measurement of the Fourier structure function of vorticity in turbulence 60
   5.1. The approach of sound scattering in turbulence 60
   5.2. Direct observation of the energy spectrum E(k) 61

6. Study of time domain spectra by sound phase shift and HWA 66
   6.1. Energy spectrum acquired by HWA 66
   6.2. The spectrum of sound phase shift fluctuation. 72
   6.3. Comparison of scaling results 77
   6.4. The spatio-temporal iso-correlation map 78

7. Moments of velocity difference ("structure functions") 80
   7.1. Hypothesis 80
   7.2. Correction of the Taylor hypothesis 81
   7.3. Third order moment curves compared between HWA, sound, and PIV. 81
   7.4. PDF of HWA and sound 86
   7.5. Extended Self Similarity (ESS) 89

8. Conclusions 91

9. Appendix A 93
   A.1. Relation between velocity and vorticity in the Fourier space 93
   A.2. Energy spectrum in the case of isotropic turbulence 93

Appendix B 94
B.1. Fourier transform in the case of axisymmetry 94
B.2. Phase shift slope with a point-like emitter and a single vortex 95

Appendix C 96
C.1. Definitions of the scales and numbers in turbulence 96

Appendix D 98
D.1. Analysis of Kraichnan’s source-term of sound 98

10. Bibliography 99
Chapter 1

Introduction

Fluid turbulence, albeit an abundant phenomenon in nature, offers a challenge in respect to observation achievements. The problem in achieving adequate observations in turbulence should become clear from this chapter, where I describe some of the flow measurement methods. Attention is made on how the idea of sound scattering evolved, in order to introduce the goals of this work. Background on turbulence is given in appendix C, or better is found in [Frisch 1995]. The starting point of the following review concerns with a common measurement in turbulence.

The choice of probe in turbulence requires some survey, since it involves a compromise between the acquisition of significant and interesting details, on one hand, and the level of confidence in the results, on the other. For example, let us consider the flow velocity correlation as function of separation distance. In order to obtain the velocity autocorrelation one can choose safely the method of two point measurement of velocity, which provides information on a single separation distance. Small separation may be limited by the physics of the intrusive elements that are used such as the wires in hot-wire anemometry (HWA). The separation distance may be varied, yet the convergence of each observation is under some doubt. Preferably the information on velocity gradient should be followed from a simultaneous measurement at multiple points. An intrusive probe of multiple elements raises the risk of considerably perturbing the flow. Thus, one is tempted to rely on Taylor’s frozen turbulence assumption and to utilize the signal of one probe in the time domain [Monin & Yaglom 1975]. When time is interpreted as distance, it is of course an easy way to obtain large amount of information on extended range of scales, but the interpretation of such information is uncertain. Additional question is then raised, whether the flow characteristic is homogeneous and isotropic. Otherwise, local and single component information is not valuable. One tends to rely on a second assumption, that at small scales turbulence is isotropic.
There is no controversy about the existence of universality in the kinetic properties of turbulence; nevertheless, the questions regarding Taylor’s hypothesis and the isotropy at small scale are considered the central questions that remain unsolved [Nelkin 1992]. When the large scale flow fluctuates around zero velocity the system is more likely in homogeneous and isotropic conditions, thus the scaling laws can be carefully examined. On that account the flow between two counter rotating plates (also known as “von Karman” swirling flow) has been suggested and gain increased interest in the last decade. It appears that in the counter rotation setup the values of turbulence intensity (indicating level of fluctuations) surpass that of the flow behind a grid in a wind tunnel of similar characteristic size. Higher turbulence intensity is associated with asymptotic universal properties, stressing the importance of such system.

Taylor’s hypothesis applies when turbulence fluctuations are much smaller than the mean flow velocity $\bar{v}$; the consequence is a relation between frequency and wavenumber via $k = \omega / \bar{v}$. H. Tennekes considered the sweeping of inertial scale eddies by large scale eddies in the case of vanishing mean flow [Tennekes 1974]. As a result the Tennekes spectrum of kinetic energy $E(\omega)$ can be matched with the Kolmogorov spectrum $E(k)dk/d\omega$ if a transformation $k = \omega / \nu'_{ms}$ is assumed, where $\nu'_{ms}$ is the characteristic large scale fluctuation (thus $\nu'_{ms}$ replaces $\bar{v}$ in the Taylor hypothesis). After some debate and numerical simulations this idea was approved [Chen & Kraichnan 1989]; however, the analysis requires that the pumping length scale (presumably the integral scale) is much larger than the Taylor microscale (the lowest scale in the inertial range), and this condition may become marginal. Experimentally, the case of vanishing mean flow can be checked in the mid plane of the counter rotation apparatus. Further, a generalization of the advection hypothesis can be stated, that for short times $t$ the corresponding spatial domain separation is $r = -<|v|>t$. This point is discussed when we concern the analysis of time domain correlation functions.

Special attention is dedicated to the third order correlation function of velocity difference, $S_3(r) = <(v_r(r_0 + r) - v_r(r_0))^3 >$, since its functional dependence is
theoretically established. Considerable work has been done in wind tunnels [Mestayer 1982; Saddoughi 1993]; however, regarding the Karman flow only the works of P.Tabeling et al treat the issue of third order correlation function [Moisy & al 1998; Zocchi & al 1994]; the measurements are performed on low temperature Helium with unique HWA system. We are interested to check a Karman flow with another fluid, and to add information on the mid plane, where average velocity vanishes and the flow isotropy is ultimate. The time domain measurements should be converted to space domain by a direct procedure. An indirect correction of Taylor's hypothesis was proposed by Pinton and Labbe [Pinton & al 1994], based on resampling (by interpolation) of the time domain signal, so that roughly the advection steps between sampling points have the same length. Thus it was easy to calculate a reconstructed spatial correlation function. My suggestion is to use a different method: to check directly the cumulative distance between each two sampling points and to sum up the results accordingly in a spatial correlation database. This procedure is more elaborate but is definitely needed in the case of vanishing mean flow.

An alternative experimental method in turbulence which requires none of the aforementioned assumptions is Particle Image Velocimetry (PIV). The intrusive elements are seeding particles immersed in the fluid and being swept by the flow. The particles function as markers of the trajectories of the flow when they are illuminated by laser pulses and pictured by a fast double frame camera. There are currently three drawbacks in the method: 1) A particle may have a different trajectory than that of the flow, especially at high acceleration points, if the densities of the fluid and the particle do not exactly match. Similarly, if the particles are very small they are infected by Brownian motion that causes them to loose the track of the flow. 2) The method achieves low accuracy in determining velocity fluctuation due to resolution limitation of the camera (for example, after resolution enhancement by interpolation an accuracy of 1% is reached on a mesh of 50x50 velocity vectors when 1M pixel camera is being used). 3) A geometry of a camera looking above the illumination plane is unattainable in the counter rotation apparatus. Furthermore, in special circumstances adding the particles is not allowed or possible. Yet, significant information on the direct spatial correlation
functions may be extracted from PIV. Also a 3D velocity map can be acquired by stereoscopic imaging, however the accuracy deteriorates. Considerable technological development should emerge before PIV amends to a satisfactory tool in turbulence. One can mention some additional techniques based on interesting effects with particles, such as Laser Doppler Velocimetry and Sound Doppler Velocimetry, and lately a method of trajectory triangulation of a single particle using a sonar [Mordant & al 2002]; these local probes we shall not discuss.

Sound is the only non intrusive means to determine the structure of a flow. The prospect of sound as a probe of turbulence was initially discussed by M.J.Lighthill [Lighthill 1952] in analysis of the sound generation by a turbulent flow. The source of sound, as was suggested, is an oscillating quadruple of the stress tensor, which on turn depends on gradients of velocity. Immediately after Lighthill it was realized by R.H. Kraichnan [Kraichnan 1953] that an oscillating quadruple can be formed by the interaction of velocity with external sound field. Kraichnan derived the general wave equation of sound in a flow, showing a source of the form $\frac{\partial^2 (v_i v_k)}{\partial x_i \partial x_k}$. He solved the problem of sound scattering specifically by looking on the dominant coupling term $v_i' v_k'$, where $v_i'$ is the fluid velocity due to sound and $v_k'$ is the usual flow velocity (differentiated from sound by its lower frequency components). In this pioneering work, Kraichnan already realized that by performing sound scattering measurement one can acquire the energy spectrum in turbulent flow [Kraichnan 1953 Eq. 5.14].

In 1959 Kallistratova performed the first sound scattering measurement of turbulence in the atmosphere [Monin & Yaglom 1975 sect. 26.3]. The scattering intensity was compared at different angles with Kraichnan’s formula presuming a Kolmogorov energy spectrum, and agreement was found. Some of the conditions of the experiment were chosen successfully: The sound wavelength of 3cm was much smaller than the transducer size of 1m$^2$ so that the beam was highly directional, and a receiver was placed at a distance of up to 140m. The scattering volume was formed by the cross section of the sound beam and the volume of detection, both collimated, thus the scattering angle could
be controlled. Due to the large distance of propagation and modulation of the sound as pulses it was possible to separate on an oscilloscope the scattering signal from the residual signal of the direct beam, thus the scattering values were recorded. Later experiments were done for meteorological purpose [for example Bass &al 1991]. In lab, similar experiment was conducted in grid turbulence in a wind tunnel by W.Baerg and W.H.Schwarz [Baerg &al 1966]. Again pulses of sound were secreted from a localized scattering volume, although the instrumentation was already more elaborated. The wavelength was varied between 0.5 and 1.5cm and the transducers of 10cm length (nearly the size of the wind tunnel nozzle) were hold across the jet at a separation of about 5m. The result showed an agreement with Kolmogorov theory. Unfortunately, the lack of accuracy in both experiments and failing to characterize the flow by other means inhibited any gain of new information about turbulence.

C.M. Ho and L.S.G Kovasny [Ho &al 1976] revealed an interesting approach concerning measurements of fluctuation in phase of sound that propagates through a turbulent jet. They estimated the Taylor scale in their flow by a hot-wire techniques. Then they looked on sound measurements at various wavelengths, and obtained the correlation half time values $T_{1/2}$ from the time correlation of phase shift. It was shown that the ratio of wavelength to the Taylor scale is an indicator for the dominant acoustic regime. When this ratio is smaller than one it is the case of geometrical acoustics, and the phase shift correlation curves are wavelength independent. When the number is larger than one, then $T_{1/2}$ depends on the square root of this ratio of wavelength to the Taylor scale. Another finding was an approval for the Taylor hypothesis based on measuring the spatio-temporal correlation between the signals of two microphones. The microphones were placed at two points along the main flow of the jet. The approval was based on drawing isocorrelation map (on normalized time and space axes) and showing the formation of circles. Only the computation ability at that time did not allow extracting still a broader knowledge that is concealed in phase shift fluctuation data.

Scattering analysis is meant as an accurate way to describe wave interaction with the medium, which includes both phase and amplitude variations, based on refractive effects,
induced sources, entrainment, and diffraction. The significance of sound scattering analysis was pointed out by A.L. Fabrikant [Fabrikant 1983] and later by F. Lund and C. Rojas [Lund & al 1989]; they realized that the scattering pattern at infinity directly concerns with the structure of a flow. Explicitly, there is a linear relation between the Fourier component of the scattering wave amplitude and the spatial and temporal Fourier transform of vorticity component normal to the plane of wave propagation. The condition for allowing this relation is to have an incident field of infinite planar wave, and to consider scattering at far field where also the flow is completely stopped. The latter condition was not obvious to workers who developed models with a velocity field decaying as $1/r$ (the distance from a center of a vortex, for example) up to infinity, not realizing that the models are unrealistic. Other approximations used in the theory such as the Born approximation, the low Mach number and incompressibility of the flow were not considered cardinal [Sakov 1993]. However, when the constraining conditions are not fulfilled there is no direct relation between the scattering amplitude and the Fourier structure function of vorticity, and then the scattering analysis becomes a problem of inverse scattering.

Oljaca et al [Oljaca & al 1998] measured sound scattering induced by a single vortex, in conditions that are close as possible to meet the direct scattering theory. The vortex was stretched in a jet of a size that was much smaller than the sound beam and much larger than the wavelength, and accordingly the detector was placed at far field. Although the workers referred to Lund’s formula in terms of vorticity they calculated the scattering amplitude from a finite region velocity field. In this way they eliminated the divergence that the theory predicts for a flow of non vanishing circulation; in fact, their calculation for a finite beam came in good agreement with the measurement (we explain this calculation farther in the theoretical chapter). In other systems of a single vortex, produced between co-rotating plates, the problem of data interpretation was encountered. Labbe and Pinton [Labbe & al 1998] demonstrated that the phase shift pattern of sound follows the precession of a single vortex. Berthet [Berthet & al 2003] used direct numerical calculations in order to predict the scattering values in Labbe’s experiment. The agreement was not perfect; moreover, Berthet missed a treatment of the beam
propagation element. A revised theory of sound scattering was seemed to be required. I think that a common mistake was made by other workers [Deroncourt & al 1998] to assume that the scattering at large angle in a turbulent flow could be analyzed by Lund’s formula. The limiting size of the beam must induce diffraction patterns (side lobes) that provide the dominant scattering signal. Besides, the experiments of sound scattering in a water tank were based on sound elements placed on the wall, not far from the flow.

The key solution for the problem of inverse scattering, i.e. gaining information on the flow from sound measurements, lays in simultaneous acquisition. In the technical aspect, we have built a unique detection system that for the first time is able to acquire simultaneously the scattering field at many angles. There is partial resemblance between our system and the detection system built by Fink and collaborators [Manneville & al 2001], which both are built around a linear array of 64 acoustic transducers. The other system relays on unfiltered analog to digital conversion at high rate, and the result is refined by utilizing a construction of time-reversal mirror that amplify the effect of the flow (via reflections of sound across the flow). Such approach essentially depends on geometrical acoustic approximation. Our system relays on analog lock-in amplifiers that filter and sample the sound signal of each channel independently. It is possible to acquire the wavefunction variation in a single pulse, by our system, to study turbulent flows. Refining the result is still possible by binding together several pulses (if one compares this system with a sonar the detection here is based on a forward scattering, and it is multi-channeled and completely coherent, i.e. it depends on a clear sinusoidal signal).

The first question that we face in this work can be phrased as follows:

- Is it visible to obtain reliable information about velocity and vorticity of a large vortex either stationary or time dependent from a resulting scattering signal of a finite width sound beam by a finite width receiver and rather close to the scattering region?

In the next chapter the theory is presented, which we developed in order to face such limitations. Inevitably, a second question arises:

- Do the predictions in the stationary flow case fall into line with the scattering data?
In chapter 3 the experimental methods are described, and in chapter 4 the answer to the questions is provided. The conclusion is based on comparison between PIV and sound scattering methods on results of the structure of a single vortex.

In chapter 5 we present results of sound scattering in turbulent flow produced between counter rotating plates. The relevant question is thus:

• Does a far field construction of the scattering data allow to obtain the exact energy spectrum E(k), and for which k values?

The conclusion is based on comparison with the Kolmogorov spectrum and with HWA measurements in parallel. The comparison with the time domain HWA data also provides a test of the Taylor hypothesis.

In chapter 6 we present the study of phase shift fluctuations in the time domain, and inquire:

• Do phase shift fluctuations reveal the energy spectrum?

The iso-correlation map (in time and space axes) is presented and the question addressed is:

• Does the Tennekes hypothesis apply?

In chapter 7 a hypothesis is raised regarding a linear relation between moments of phase shift difference and moments of velocity difference. The principle question is:

• Do phase shift fluctuations provide a way to acquire the statistical moments (structure functions) of velocity difference?

The conclusions are derived from comparison with HWA and PIV measurements and additional comparison between two measurement planes: the middle plane where the average velocity vanishes and a shifted plane closer to one of the plates.
Chapter 2

Theory of ultrasound scattering by flow

2.1. Sound scattering by a finite width beam.

Following a derivation of an equation for sound scattering and refraction by a flow mostly due to [Kraichnan 1953] one gets in the first approximation of a small Mach number, \( M = \frac{v}{c} \), the following equation for the sound scattering caused by the velocity field alone:

\[
\Delta \Psi + k_0^2 \Psi = -2 \rho_0 \frac{\partial^2 (v' \cdot v_G)}{\partial x_i \partial x_a}
\]  

(2.1)

Here \( \Psi \) is the complex wave function that represents the sound pressure oscillations generating at frequency \( \omega \) by the emitter, \( k_0 = \frac{\omega}{c} \) is the sound wave number, \( \vec{v} \) and \( \vec{v}' \) are the flow velocity and the velocity oscillations due to sound propagation, respectively, and \( \rho_0 \) is the uniform fluid density. Eq.(2.1) was obtained in the approximation where low frequency sound generated by the flow and the sound attenuation due to viscosity were disregarded. Further, we assume 2D symmetry for simplicity, and due to the fact that the sound frequency is much larger than any frequency in the flow a quasi-stationary case is considered. The influence of, e.g., moving vortices in the flow shows up in a frequency shift (Doppler shift) relatively to the incident wave frequency. The former usually is smaller or comparable with characteristic frequency of fluid flow and is not considered here. In order to evaluate Eq.(2.1), we choose the axis \( x \) along a local incident wavefront direction, thus \( \vec{v}' \parallel \hat{x}_i \), and expand the source term (see appendix D). There is no influence of the curvature in the wavefront direction on the derivatives within the first order in the Mach number approximation [Landau & Lifschitz 1987]. Then taking into account that the wavelength is the smallest scale in the flow one gets for an incompressible flow the following equation (in a plane wave approximation)

\[
\Delta \Psi + k_0^2 \Psi = \left[ 2k_0^2 \vec{v} \cdot \hat{x} - 2ik_0 \frac{\partial (\vec{v} \cdot \hat{x})}{\partial x} \right] \Psi / c,
\]  

(2.2)
where \( \hat{x}_i \) is replaced by \( \hat{x} \) and is used to define the local ray direction.

One can define the sound scattering field as \( \Psi_{\text{scat}} = \Psi - \Psi_{\text{rest}} \), where \( \Psi_{\text{rest}} \) is the complex wave function in the absence of a flow that satisfies the following wave equation without a source term:

\[
\Delta \Psi_{\text{rest}} + k_0^2 \Psi_{\text{rest}} = 0. \tag{2.3}
\]

By applying further the Green function method the solution of the scattering problem is obtained for a two-dimensional geometry (based on the Born approximation and the single scattering limit) in the following form (in \( k_0 |r - r'| \gg 1 \) approximation):

\[
\Psi_{\text{scat}} = \frac{1}{c} \exp(i \pi / 4) \int d^2r' (k_0^2 \nabla_x - ik_0 \frac{\partial}{\partial x}) \exp(ik_0 |\vec{r} - \vec{r}'|) \Psi_{\text{rest}} \sqrt{2 \pi k_0 |\vec{r} - \vec{r}'|} \tag{2.4}
\]

In a far-field the solution can be drastically simplified and reduced to the analytical relation between the scattering field and the Fourier transform either of the velocity or the vorticity fields. Indeed, in the far-field \( |\vec{r} - \vec{r}'| \gg |\vec{r} | - \vec{r} \cdot \vec{r}' \), and the integral in Eq.(2.4) becomes the Fourier transform of the velocity and the velocity gradient fields. The variable in Fourier domain is the scattering wave vector \( \hat{k}_s = k_0 (\hat{r} - \hat{x}) \) and \( |\hat{k}_s| = 2k_0 \sin \theta / 2 \), where \( \hat{r} \) is the unit vector from the center of the scattering region toward the detector. The Fourier transform in 2D is defined as

\[
F_{k_s} \{v_x \} = \frac{1}{(2\pi)^2} \int d^2r' \exp(-ik_s \cdot r') v_x (r').
\]

Taking into account that \( F_{k_s} \{\frac{\partial \nu}{\partial x} \} = i(\hat{k}_s \cdot \hat{x}) F_{k_s} \{v_x \} = ik_0 (\cos \theta - 1) F_{k_s} \{v_x \} \), one obtains in the far-field limit

\[
\Psi_{\text{scat}} = \frac{1}{c} \frac{(2\pi k_0)^2 \exp(i \pi / 4)}{\sqrt{2 \pi k_0 r}} \cos \theta \ F_{k_s} \{ \Psi_{\text{rest}} \}, \tag{2.5}
\]

where \( \cos \theta = \hat{r} \cdot \hat{x} \) and \( \Psi_{\text{rest}} = \Psi_{\text{rest}} \exp(-ik_0 x') \). We present in Eq.(2.5) the scattering field as a convolution of the Fourier transforms of the velocity field and the known beam function, \( F_{k_s} \{\Psi_{\text{rest}}\} \). Only in an ideal case of an infinite planar wave front, when
\( \Psi_{rest} = 1 \), there exists a simple relation between the Fourier transforms of the velocity and the vorticity fields (see appendix A.1.):

\[
F_{ks} \{v_s\} = \frac{i(\hat{y} \cdot \hat{k}_s)}{k_s^2} F_{ks} \{(\nabla \times v)_s\} = \frac{i}{2k_0} \cot(\theta/2) F_{ks} \{(\nabla \times v)_s\}
\] (2.6)

which leads to a direct linear relation between the sound scattering field and the Fourier transform of the vorticity field as first pointed out by Lund and Rojas [Lund & al 1989].

This general approach can be applied in a case of a single rigid body rotation vortex with a core azimuthal velocity distribution as

\[
v_i(r) = \Omega r, \quad \text{for} \quad r \leq r_0
\] (2.7)

and outside the core up to the cell wall the azimuthal velocity component decays as

\[
v_i(r) = \frac{\Omega r_0^2}{r_c^2 - r_0^2} (-r + r_c^2/r), \quad \text{for} \quad r_0 < r < r_c
\] (2.8)

where \( \Omega \) is the angular speed, \( r_c \) and \( r_0 \) are the cell and the core radii, respectively (see Fig. 2:1).

---

**Fig. 2:1. Geometry of cell, flow, and sound scattering.**
In the axisymmetric case one gets the following result for the Fourier transform of the x velocity component, $v_x$, based on the flow model Eqs.(2.7)(2.8):

\[
F_{k_s}\{v_x\} = -\frac{i}{2\pi} \cos(\theta/2)\Omega \frac{2r_0^2}{k_s^2(r_s^2 - r_0^2)}[J_i(k_s r_e) r_e - J_i(k_s r_x) r_0],
\]

where $J_i(x)$ is the Bessel function, which was introduced via

\[
F_{k_s}\{v_x\} = \frac{-i(\hat{y} \cdot \hat{k}_s)}{2\pi k_s} \int r^0 v_x(r')J_i(k_s r')dr'.
\]

The expression (2.9) exhibits even for the unbounded beam width and detector length two maxima in the scattering signal at some value of $k_s$ due to a final size of the flow region, $r_e$. It is clear from Eq.(2.9) that in a general case the location of the maxima depends on two parameters, $r_0$ and $r_e$. In the case when $r_0/r_e \ll 1$, the peaks are located at $k_s r_e = 1.11\pi$. This value alters by change of $r_0$. Calculations based on Eq.(2.9) give the following results (for $r_e = 145$ mm): at $r_0 = 6$ mm the peak position is at $k_s r_e = 1.11\pi$, at $r_0 = 20$ mm the value is $k_s r_e = 1.09\pi$, at $r_0 = 50$ mm the expected value is $k_s r_e = 1.02\pi$, but at $r_0 = 100$ mm the location value is already shifted to $k_s r_e = 0.87\pi$.

The peak height, $H$, also provides useful information about the flow. Indeed, in the case of $r_0/r_e \ll 1$ and of the infinite beam width, $H$ is proportional to $\Omega$, to the core circulation, $\Gamma = 2\pi\Omega r_0^2$, and independent of $k_0$ (or the sound frequency). The dependence on $\Gamma$ and independence of $k_0$ are rather non-trivial results taking into account rather complicated functional dependence of $F_{k_s}\{v_x\}$ on the parameters $r_0$, $r_e$, and $k_0$.

In the limit $r_e \to \infty$ the peak locations approaches zero angle asymptotically but the value of the amplitude of the Fourier transform of the velocity at $\theta = 0$ remains zero. In the limit of $k_s r_e \ll 1$ and at $r_0 \ll r_e$ one can derive an asymptotic expression for the Bessel functions in Eq.(2.9) and get $F_{k_s}\{v_x\} \mid_{k_s r_e \to 0} = -\frac{a_0}{\pi^2} r_0^2 k_0^2 \sin \theta / 2$. From this expression it is easy to see that the amplitude at small angles increases proportionally to $\Omega$, $\Gamma$, and $k_0$, and approaches zero at $\theta \to 0$. 

16
For a finite width beam (or emitter) or a finite width detector the Fourier transform of the scattering wavefunction from Eq.(2.5) can be expressed as a convolution $\tilde{\Psi}_{\text{scat}}(\theta) \sim F_{k_y} \{ v_y \} \ast F_{k_y} \{ \tilde{\Psi}_{\text{rest}} \}$. In a case of the finite width, $d$, (either of a beam or a detector whatever is the smallest) let us consider first the Gaussian beam model. The beam function is defined as $\tilde{\Psi}_{\text{rest}} = \exp(-y^2/[2(d/2)^2])$. For $d/r_c \gg 1$ the behavior of the peak height here is similar to the infinite width beam considered above.

The location of the peaks in a finite width Gaussian beam depends on the beam width: at $d=50\text{mm}$ the peak position is at $k_sd \simeq 0.94\pi$ (at $r_0/d \ll 1$). This is expected since roughly the first antisymmetric mode $\sin(y\pi/d)$ is the main contributor to the convolution. When $r_0$ increases this relation is changed: according to numerical calculation at $r_0=10\text{mm}$ the peaks are located at $k_sd \simeq 0.92\pi$, and at $r_0=50\text{mm}$ they are found at $k_sd \simeq 0.71\pi$. The dependence of the peak heights on the parameters for a finite width beam is more elaborate, since here three characteristic lengths exist in the problem: $r_0$, $r_c$, and $d$. In spite of this fact the approximate scaling law was found from numerical calculations. It follows that $H$ is independent of $k_0$ and proportional to $\Gamma$ with the scaling function $g(x)$ (where $x=r_0/d$) that is different for two regions of small and large beam width: (a) $g(x)=125x^{1.2}$ for $x>1$, and (b) $g(x)=66+66x^2$ for $x<1$.

In the case of a finite rectangular beam (or a finite detector whatever is the smallest) the peak location is defined by $k_sd$ as in a finite Gaussian beam considered above. At $r_0/d \ll 1$ one gets $k_sd \simeq 1.52\pi$ (e.g. for $d=50\text{mm}$). For larger values of $r_0$ the peak location is shifted towards smaller values. So, at $r_0=10\text{mm}$ the estimated peak position is at $k_sd \simeq 1.48\pi$, and at $r_0=20$ and 50 mm one gets $1.39\pi$ and $1.34\pi$, respectively. The peak height similar to the case of the Gaussian finite width beam, is independent of $k_0$ and proportional to $\Gamma$ with the scaling function $g(x)$ that is slightly different from the former case. It is determined for the range of arguments values $0.01 < x < 4$ at $d>30\text{mm}$.
and $r=145\text{mm}$ and has the following form: (a) $g(x) = 205x^{1.2}$ for $x > 0.5$, and (b) $g(x) = 63 + 103x^2$ for $x < 0.5$.

Fig. 2:2. Numerical simulations of the Fourier transform of the velocity field of a single rigid body rotation vortex flow ($r_0 = 5\text{mm}$) for a finite width beam, $d$, compared to an infinite beam: (a) $d=120\text{mm}$, (b) $d=60\text{mm}$, (c) $d=20\text{mm}$. Solid line: a finite width beam; dash-dotted line: infinite width beam.
In order to demonstrate the influence of a finite beam width on the scattering signal from a single vortex with a core radius, \( r_0 \), we present in Fig. 2:2 the results of numerical simulations for the Fourier structure functions of the velocity field for the vortex of \( r_0 = 5 \text{mm} \) and three beam widths of 20, 60, and 120 mm (but all of them smaller than the cell size, \( r_c = 145 \text{mm} \)) compared with an infinite beam width for the same cell size. Only for \( d = 120 \text{mm} \) the wave number of the structure function peak location becomes close to the peak location for the infinite width beam (see Fig. 2:2a). The wave number of the peak location in the infinite width beam case is defined by the cell size (see explanation above after Eq.(2.9)). One can see that in spite of the fact that \( d \gg r_0 \), the finite width of the beam drastically alters the resulting scattering signal. Thus in order to get correct results on the structure functions of the velocity and vorticity fields one should use either a beam width exceeding the flow size, \( r_c \), or to perform a deconvolution based on a provided constrain, which fills out the missing information about the velocity field outside the beam extent.

### 2.2. Alternative approach to find velocity and circulation: phase shift slope in a forward scattering.

While the study of scattering data requires to extract from sound signals both the amplitude and the phase variations, one can find information about the velocity field and the circulation from the phase shift. We consider two cases in a phase shift analysis. In the first case of a large emitter, rays remain parallel in propagation, and the phase shift as function of \( y \) (assuming it can be measured avoiding interferences between the paths) can be defined as:

\[
\phi(y) = (k_0 / c) \int_{x}^{y} v(x', y')dx'
\]  

(2.10)

Lindsay has found [Lindsay 1948] that the phase shift induced by a point vortex has a linear dependence on an angle. This result can be obtained if we consider rays passing outside the core, and assume a large emitter at \( x \to \infty \) and velocity profile \( 1/r \), namely:
\[ v_x = \frac{\Omega r_0^2 y}{x^2 + y^2} \]  

(2.11)

Phase shift is measured on a screen at \( x = r_c \), with \( \theta = \tan^{-1}(y/r) \), then the slope can be found, outside the singularity at \( \theta = 0 \), with the following constant value:

\[ \frac{\partial \phi}{\partial \theta} = \frac{\partial}{\partial \theta} \left[ \int_{-\infty}^{r_c} \frac{k_0 v}{c} dx \right] \]  

(2.12)

Our idea is to look in particularly on rays passing near the center of the vortex at \( \theta = 0 \) and try to get information on the flow from the phase shift slope. For an infinite source at \( x = -r_c \) and detectors at \( x = r_c \):

\[ \frac{\partial \phi}{\partial \theta} \bigg|_{\theta \to 0} = \frac{k_r r_c}{c} \int_{-r_c}^{r_c} \frac{\partial v}{\partial y} dx \quad \text{at} \quad y \to 0. \]  

(2.13)

Thus, substituting the velocity field projected on \( x \) via Eqs.(2.7)-(2.8), one obtains the phase shift slope as

\[ \frac{\partial \phi}{\partial \theta} \bigg|_{\theta \to 0} = \frac{k_r r_c}{c} 4\Omega_0(1 - \frac{r_0}{r_c + r_0}). \]  

(2.14)

Comparing Eq.(2.14) and Eq.(2.12) we find that a rigid body rotation core induces a larger slope by an order \( r_c/r_0 \) compared with the exterior of the vortex. Thus for a point vortex at \( \theta = 0 \) there is a step in the phase shift, having an infinite slope, revealed therefore as the Berry's phase [Berry \& al 1980].

A total change in the phase shift between two points on the acoustic screen can be calculated by a loop integral over the velocity field. Since a velocity along the detector and the emitter is zero, there is a simple relation between the circulation and the step in the phase shift followed from Eq.(2.10) [Sonin 1997]:

\[ \Gamma = \oint \vec{v} \cdot d\vec{l} = \frac{c}{k_0}(\Delta \phi_{(2)} - \Delta \phi_{(1)}), \]  

(2.15)

where \( \Delta \phi_{(1)} \) and \( \Delta \phi_{(2)} \) are the phase shift differences on two sides of the detector, respectively.

In the second case we consider rays diverging out of a single point on a small emitter (and there is an axisymmetric single vortex). We can show that the phase shift slope,
calculated from the phase shift due to a vortex at the cell center between a detector placed
at $x = r_c$, and sound emitter located at $(x, y) = (-r_c, 0)$, can be written as

$$\frac{\partial \phi}{\partial \theta} \bigg|_{\theta \to 0} = \frac{k_0 r_c}{c} \int_0^r \frac{\nu_x(r)}{r} dr$$

(2.16)

(see appendix B.2.). The angle $\theta$ is measured between the radial vector from the center of
the vortex and the beam direction. Substituting the expression for the velocity profile
Eqs.(2.7)-(2.8) into Eq.(2.16) we obtain:

$$\frac{\partial \phi}{\partial \theta} \bigg|_{\theta \to 0} = \frac{k_0 r_c}{c} 2\Omega (1-\frac{r_0}{r_c + r_0})$$

(2.17)

that is equal exactly a half of the result for a large emitter.

2.3. Construction of a far field wave function from the near field measurements.

Probably, the most severe limitation of the theory to be applicable to an experiment is
the far-field approximation. In this respect the relevant question is how to extrapolate
sound scattering measurements obtained at a distance not far away from a scattering
region into a far-field region to be compared with the theory and to reliably extract
information about the vorticity structure function.

A far-field construction of the scattering field from the acoustic field (either pressure
or scattering wave function) given at a certain plane in 3D case and at a certain line in 2D
case, is based on the mathematical description of a Huygens principle typically used to
describe radiation from a curved surface. This problem is similar to the problem of
diffraction by a thin screen of finite dimensions (or for a finite dimension wave beam)
[Marston &al 1992]. Thus our goal is to consider the propagation of a wave into
unbounded source-free half-space $x \geq 0$, when certain conditions in the initial plane $x=0$
are specified. Then the scattering field in 2D at the location $x \geq 0$ can be defined from the
following Rayleigh-Sommerfeld integral [Marston &al 1992]:

21
2.4. Conclusion

The previous view regarding vorticity as a scatterer of sound fails if we try to consider spots in the flow that are not illuminated by the sound beam. Our theoretical suggestion is to use the beam function $\Psi_{\text{rest}}$ in a product with velocity (as in Eq.(2.5)) in order to describe the scattering in a similar formalism as for infinite width beam.
The scattering wave is in first part due to induced phase shifts as result from entrainment of the wave by the flow. The second part (corresponding to the second term in Eq.(2.4) ) is due to the spatial derivative of velocity in the beam direction, which becomes negligible as the frequency of sound increases. The analysis of the Huygens construction of the far field is aimed to compensate for the diffraction of the corresponding distortions of the wave. In this construction method we obtain the acoustic scattering signal in far field from the known sound distribution in any intermediate plane.

We predict some results of the phase shift slope, and of the scattering amplitude in the case of a rigid body rotation vortex. These predictions are verified in the experimental sections.
Chapter 3

**Experimental set-up and measuring techniques**

The experiments were performed in a flow, which was produced inside a closed cylinder either with one rotating disk or in the gap between two coaxial either co- or contra-rotating disks (the so-called von Karman swirling flow). As detection techniques we use ultrasound scattering, particle image velocimetry (PIV), and hot-wire anemometry (HWA) to directly measure velocity field.

![Experimental set-up diagram](image_url)

**Fig. 3:1.** Experimental set-up: (1) rotating plates, (2) light sheet and scattering plane, (3) PIV camera with Scheimpflug optic arrangement, (4) acoustic emitter, (5) main signal pulse, (6) transmit switch, (7) detector array, (8) 64 preamplifiers, (9) 64 lock-in amplifier, (10) reference signals, (11) sample and hold trigger, (12) acquisition cards (128 channels), (13) hot-wire probe.
3.1. Hydrodynamic set-up.

The hydrodynamic cell consists of a vertical Plexiglass (Perspex) cylinder of 29 cm inside diameter and 32 cm in height (see Fig. 3:1). A swirling water flow is produced between two rotating plates driven independently by two motors with a maximum continuous torque of 13Nm. The motors are brushless sinusoidal ones, currently controlled by a motion card (PCI-7344 from National Instruments) via optical encoders, reaching constant velocity within 0.1-0.3% error (although for part of the work on single vortex only velocity mode drivers with 0.5-1.0% error were available). Each shaft is centered and is fixed by two bearings and sealed with Uniten rotating rings. The base of the device is suspended on shock absorbers and the shafts are isolated from vibration by the motor couplings (Rotex GS designed for servo systems). The acoustic components (emitters and a receiver array) are mounted flush to avoid obstacles to the flow (see Fig. 2:1). The cell is completely filled with deionized water (with addition of 10ppm of surfactant when PIV measurements are required). The temperature is stabilized at 25 deg Celsius using heat exchange with the top and bottom covers of the cell, made of stainless steel, in circulation with a Lauda temperature control.

The rotation speed is limited in the case of counter rotation mode by cavitation starting mildly at 420 rpm (Re=1.5·10⁶) and at 540 rpm creating large bubbles that mask entirely the passage of ultrasound. In the case of co-rotation mode the speed limit is due to a suction of air and formation of a wide filament of bubbles at the center of the vortex (at 600 rpm). After filling the cell cavitation starts even at 300 rpm. The air dissolved in the liquid is gradually removed by collecting the bubbles formed by cavitation. Bubbles that are trapped in the upper plates are shaken out by switching the direction of rotation. Small bubbles tend to coagulate in the co-rotation mode. Such techniques are used during several days to reach low dose of dissolved air in the working liquid. Another less effective approach is to add soap to the liquid and thus to reduce the size of the bubbles and reduce their damage.
Several stirring configurations have been used (see Fig. 3:2). In the single vortex mode we can list: (i) Two stirring plates of 100 mm diameter are positioned 100 mm apart, each plate has four blades of 20 mm width and 5 mm thickness; (ii) the same kind of plates with 40mm diameter and 4mm blades thickness; (iii) Upper plate is a smooth metal disk of 280 mm diameter, a lower plate is absent, and only 25 mm diameter shaft is present with upper plane located at 50 mm from the bottom of the cell; (iv) Two elongated shafts of 25 mm diameter are set 86 mm apart, grinded at the edge to a shape of one blade of 12 mm width and 6 mm thickness. In the study of turbulence we used a different configuration: (v) The upper and lower plates are 280mm (in diameter) metal disks, set 205mm apart, with 4 triangular blades (shallow at the center and 3cm high at the circumference) of 5mm thickness, which are covered by rims of 5cm height in order to avoid intense friction at the wall. Almost similar configuration was used by [Cadot &al 1997].

Fig. 3:2. The various stirring plates used to produce flow. Left to right: 25mm, 40mm, 100mm, and 280mm diameter. The latter is shown inside the cell (shown in final configuration, the emitter is seen on left and the detector array is hardly seen on the right, upper plate is removed).

We define the Reynolds number according to

\[
\text{Re} = \frac{2\Omega R^2}{\nu},
\]

where \( R \) is the radius of plates, \( \Omega \) is the rotation speed in rad/s, and \( \nu \) is the kinematic viscosity.
The injected power to the flow was evaluated via the current delivered to the motors, after subtracting the idle current, and the results are shown in Fig. 3:3. A comparison between the different modes attributed in Fig. 3:3 indicates that most of the dissipation occur in the bulk of flow. The dotted line denotes a prediction (up to a factor) based on dimensional analysis. In the counter rotation mode (with plates of radius $R=0.14\text{m}$) we find the fitted injection rate as $Q_i = 0.12\rho_0\Omega^3 R^2$. For comparison, O.Cadot et al [Cadot \\& al 1997] found in their setup $Q_i = 0.4\rho_0\Omega^3 R^5$, and their distance between the plates is 50% longer than ours. The heating rate of the fluid in Cadot's experiment was $Q_D = 0.75Q_i$; and we obtained by temperature measurements that roughly the heating rate is the same as the injection rate, $Q_D \approx (1\pm0.2)Q_i$ (after checking that the temperature elevation rate is uniform all over the cell). The part of dissipation that corresponded to pressure fluctuation in Cadot's experiment was $Q \approx 0.6Q_i$. We assume in our setup $Q = 0.6Q_i$, and accordingly expect that the part of dissipation revealed in analysis of macroscale velocity fluctuations (in counter rotation flow) should be around:

$$\varepsilon = \frac{0.6Q_i}{\rho_0V} \approx 0.015\Omega^3 R^2.$$  

(assuming uniform dissipation over a volume $V=14$ liters of the fluid between the plates).

**Fig. 3:3.** The total power injected by the motors to the flow (in Watts units). The fitted line is based on dimensional analysis prediction.
3.2. Ultrasound probe

3.2.1. General scheme

The measurement plane where the emitter and the detector array are placed is chosen at the middle between the disks (except for cases that we explicitly mention where the plane was shifted by translating both disks and building extension to the cell). The sound detects the velocity field in the plane, or complimentarily, the vorticity field perpendicular to that plane. Ultrasound pulses are sent by a specially designed emitter (see below) and detected by a linear array of 64 acoustic detectors with 1mm spacing and 64x10mm overall active area (from Blatek). The acquisition system is built in a heterodyne scheme meaning that 64 lock-in amplifiers are utilized on the incoming analog signals. The outgoing low frequency signals are passed to sample and hold components, which integrate the signal of each pulse and hold the value to be scanned by two PC acquisition cards (National Instruments pci6071) recording 128 channels, hence providing amplitude and phase components of the signal for 64 acoustic channels. The lock-in amplifiers were compared to a SR844 RF lock-in amplifier (Stanford Research Systems) and were found linear and reliable. The acquisition cards also function as controllers of timing with precision of 0.05µs. The detector and electronics are optimized for 5.5MHz. Each of 64 preamplifiers is based on VCA2612 chips from Texas Instruments, and it is built directly around the detector. The entire circuitry consists of about 8,000 components. A special care was taken to block electromagnetic interference emitted from the motors: (i) installing filters on the motors and the power supply; (ii) the circuitry and the emitter cable are shielded in a triplex scheme where the outer shell is connected to chassis ground, the intermediate shell connected to a one point analog ground. We are able to measure amplitude and phase of the sound pressure simultaneously at 64 positions, at various Doppler frequencies. The required transmitter activating signal is only 30mVrms (on certain piezocrystals), where the response of sound picked after propagation and detection is around 0.5Vrms on the acquisition card.

The time sequence of the measurement process is implemented with two National Instruments pci6071 cards installed on a PC running on Labview, and it goes as follows:
A software key initiates once the sequence, in which a rectangular pulse is sent repeatedly and activates a switch that connects the main signal to the acoustic emitter. The pulse repetition rate is 550µs and its duration 18µs. The main signal is sinusoidal with implied frequency between 450KHz and 7MHz (3.3mm and 0.2mm in wavelengths, respectively, for water). The emitted sinusoidal signal must remain coherent between pulses and this will not be provided in burst mode of some function generators. The rising edge of the pulse triggers additional three clocks (two event clocks are available in each card). 1) A sample pulse is sent to engage the sample and hold components in the 64 lock-in amplifiers. The pulse width is 8µs, which defines the integration time; delay time of 204µs is chosen according to the acoustic propagation time with additional 5µs to allow for geometrical delays, time shifts and sound from secondary sources to arrive on the detectors. The end of the sample pulse triggers the acquisition scan by the cards. 2) A gain variation pulse is sent to the preamplifier during the sample pulse and 1µs before in order to increase the gain to a preset maximum value. Such step is taken to decrease the chance for a preamplifier to saturate and hang during the sample time in response to sudden strong noise (e.g. reflection of side pointing beam from the wall). 3) A clear pulse is sent to reset the sample and hold components by opening a path to discharge their capacitors. The discharge starts after that sufficient time is allowed for the acquisition cards to scan the channels (scan rate of 500KHz was used, and a waiting time was set assuming the cards work in sequence). The discharge time was set to 40µs (twenty times the characteristic discharge period).

In the case of stationary flow the absolute phase shift was extracted, and in order to avoid errors by some arbitrary offset in the sampled signal a strobe mechanism was utilized (but not for the turbulent flow measurements). The frequency of the two reference signals, which enter the product detectors in the lock-in amplifiers, was shifted by 100Hz in relation to the main signal (coherence remains since the three function generators were connected on the same 10MHz time-base).

The raw data is recorded in the form of I and Q values, which mean the integrated (over the pulse time) products of the pulse signal with the 0 deg and 90 deg references, respectively. The direct hashing of the data diverges between stationary flow and turbulent flow analysis. In the stationary flow case the data processing provides the
amplitude and phase of sound based Fourier analysis in time of the pulse series. The center frequency of the peak in the Fourier domain is 100Hz according to the detuning between the signal generators (see Fig. 3:4). A summing window of about 100Hz is used, which is enough to include the observed frequency bandwidth.

In the case of turbulent flow only the fluctuations in the amplitude and phase are obtained, based on the I and Q values of each pulse (see Fig. 3:5). The term I+iQ is interpreted as the complex wavefunction fluctuation since the signal is mainly composed of a single Fourier component (the Doppler shifts almost vanishes, as discussed below). Thus we track amplitude and phase variations at a rate of 1.8KHz. The scattering fluctuation values are normalized according to the mean signal modulus of the pulses. Another mode of measurement was tested, in which only 4 detector elements are activated, and the sampling rate reaches 20.8KHz. Due to increased noise pickup at elevated frequencies there is no much benefit in the latter mode. Therefore, all the results presented on turbulence are based on 62 channels of data looking as in Fig. 3:5.

The sequence of pulses is not synchronized with any of the function generators hence an initial arbitrary phase has to be resolved. In addition, since Labview on a PC is not a real-time software it is not promised that the two cards begin acquisition in the same pulse. The arbitrary phases are resolved by sacrificing two channels in each card. Thus, 62 lock-in amplifiers are used for acoustic detectors and another two are connected to the main signal (attenuated). During analysis the phase of the latter is subtracted from the former depending on which card the lock-in amplifier is sampled.

The Doppler shift in frequency should vanish in a steady flow since there are no moving sources (the frequency of detected signal is not the same as the wavelength that varies locally). However, if the large scale motion contains small flow structures that act as secondary scattering sources [Baudet & al 1999] (or if any scatterer exists in the fluid) then a secondary peak with Doppler shift is expected at \( \Delta f = k \cdot \nabla / 2\pi \) [Baudet & al] (depending on the average velocity and the scattering wavevector). Such peak was identified in the frequency domain of our observed far field scattering pattern for a vortex produced between large co-rotating plates (280mm). The peak magnitude was found 10^{1.5} lower than the non shifted peak and was located at 100Hz per 1 rev/sec rotation speed at wavenumber of 1mm^{-1} (and linearly dependent on the wavenumber via scattering angle).
Fig. 3:4. Direct I and Q signals of one acoustic channel in 100Hz detuned mode. (a) fluid at rest, (b) co-rotation flow at 5rev/s, (c) counter rotation flow at 5rev/s. (1.8KHz pulse repetition rate).

Fig. 3:5. Direct I and Q signals of one acoustic channel in tuned mode. (a) fluid at rest, (b) noise by electromagnetic interference of the motors, (c) co-rotation flow at 5rev/s, (d) counter rotation flow at 5rev/s.
In the counter rotation case a secondary peak with Doppler shift was not observed owing to the small average velocity. Owing to the Nyquist rule, high resolution in time excludes accuracy in frequency, thus we chose to focus on a single frequency component of sound.

Significant part of the efforts in building the probe was devoted to design a suitable acoustic emitter. The important parameters are the length of the transducer in the axis parallel to the measurement plane, and the smoothness of the phase and amplitude. A long transducer is necessary to produce a beam of parallel rays, to minimize the effect of side lobes, and to reduce the edge diffraction on the detector. The transducer lengths are varied between 3mm and 145mm. The transducer width is around 10mm, close to the width of the detector and determines the thickness of a layer, in which the flow is effectively averaged.

A composite piezoelectric material was used in order to decrease lamb waves distortion. However, by using piezo-composite in the largest sizes a smooth phase still could be achieved but not the amplitude. The following types of transducers were built:

(T1) This type was used for the transducers of 3.5mm and 10mm lengths. A piezoelectric plate (SPC-Y02) was covered by 4mm flat lens of Perspex and matched by conductive epoxy (Circuitworks) that was used also for wiring. The backing was air.

(T2) A piezoelectric plate (PZ34 from Ferroperm) was covered by a flat lens made of perspex with groves for the soldered wires. The aperture size was 35mm, and the backing was air.

(T3) Two or three pieces of PZ34 plates were adjusted together side-by-side in a frame to make a length of 96mm and 145mm, respectively. The backing was air, and the cover was a thin layer of epoxy. Wires were soldered on the edges.

(T4) A better transducer compared with T3 was built from polarized PVDF, of 125X10mm an active area, mounted on Balsa wood and covered by a thin, conductive layer of epoxy and a thin layer of Acrylic.

(T5) Comparable transducer to T4 was built of C6 piezoelectric material from Fuji-Ceramics, of 100X10mm size of which the active length is 80mm according to the size of the Perspex lens. The construction involves extensions at the two sides of the flat lens, which allow for a gap for the front soldered wires at the spare length of the piezoelectric
plate. The extensions are shaped to reduce the volume of the cavity between the emitter and the round wall, so the flow is almost unperturbed.

In all types of the emitters the enclosure was made of black Delrin, with 30dB/cm absorption. Types T1, T4 and T5 were leveled with the round wall. Types T4 and mostly T5 were used in measurements of turbulent flows.

3.2.2. Calibration

Calibration of the response of the detector array on each acoustic element to a plane sound wave was accomplished by moving a small emitter (19x6 mm$^2$ at 2.5 Mhz from Automation Industries) slowly and precisely, paralleled to the array and taking measurements in steps of 1mm. Summation of the complex wave function over all steps yielded the response to an ideal large emitter.

3.3. Particle Image Velocimetry (PIV)

3.3.1 Measurement scheme in single vortex mode

The standard part of the PIV system was built by Oxford Lasers. A double exposure 1Mb CCD camera and a 15mJ pulse infrared laser providing light sheet are used to measure the velocity field of the flow in two dimensions, with particles as markers [Raffel 1998]. The seeding particles are 20µm Polyamide from Dantec, dissolved in a 50% mixture with a surfactant (PolyOxyethylene-Sorbitan monolaurate from Sigma). The plane of the velocity measurement is chosen to overlap the acoustic measurement plane. The analysis in the single vortex case is based on cross correlation between images by Fast Fourier transformation (FFT) of divided regions in the frame (usually by 32X32 pixels). Analysis of each region provides one velocity vector, but the algorithm can take overlapping regions up to 75%, so that for regions of 32X32 pixels from a 1Mb pixels camera a mesh of 128X128 vectors is obtained. In the implementation, the pixel equivalent size was 80X210µm, the laser pulse duration was set to 60µs and the pulse separation was set to 2130µs. Thus, correlation between consecutive frames could fetch reliably velocities.
between 0.1-1.1m/s (in a worse case, using 32X32 pixels). There is a tradeoff between mesh resolution and velocity accuracy and limits. Our analysis expands the velocity limit by combining results of analysis with different area sizes of correlation regions. Thus, we limited our search to low velocity gradients.

The system was modified to use it in the narrow field of view available between the rotating plates. Since the design of the experimental set-up (in two disks configuration) does not allow the camera to look right above the light sheet plane, we were accounted with three problems. (i) The view angle should minimize pickup reflections from the wall. (ii) The converging effect of the circular wall of the water filled cell as well as reduced optical clarity can be corrected by using windows made of flat pockets of Perspex filled with water outside the cell. For reducing reflections separate windows for the laser sheet and the camera were found preferable. (iii) Focusing on a plane in a perspective view should use the Scheimpflug condition [Bass & al 1995], in a way known as a perspective control. The perspective control requires breaking the alignment between the camera and the lens so that the plane of the CCD (image plane) meets the principle plane of the lens at the same offset from the optical axis as the line where the object plane meets with the other principle plane of the lens. The required tilting of the lens comes on the expense of reduced light intensity entering the CCD camera. In order to increase the light intensity we used two lenses. The tilted lens chosen was Mamiya RZ67 75mm/4.5 short barrel lens, capable for the perspective control. On the digital camera, we fixed a micro Nikkor-Nikon 50mm/1:1.4 lens through a F-C mount, which was shorten down to 11mm. This configuration allows us to focus (up to some visibility of particles) on almost 15cm of the plane from 30cm distance, subject to setting obtuse viewing angle. The quality of images, however, was not sufficient to obtain the instantaneous field but rather was used to collect the average velocity field in time.

3.3.2. Measurement Scheme in counter rotation mode

In a late stage of the project we were equipped with a green light 15mJ pulse laser (532nm), which improves detection of seeding particles by a CCD camera by almost one order of magnitude compared with the infrared (808nm) laser. A standard TV zoom lens
is used with aperture set to small opening, thus a fully focused 100X60mm image is obtained at a 45° view angle. The separation time between pulses is set to 1000µs for rotation speeds up to 2rev/sec and 500µs at higher speeds, pixel size is found 120µmX60µm. The field of flow between counter rotating plates of 280mm with blades and rims is acquired in the shifted plane h=-30mm (relative to the middle plane between the plates). The image analysis is based on a multipass algorithm [Segre 2004], which involves successive refinement of correlation zones and determination of their position during iteration. This work provides instantaneous fields of 31X42 validated velocity vectors per frame.

3.3.3. **Calibration**

A calibration chart occasionally planted in the illuminated plane is photographed and the scales are fed into the PIV software in terms of $x$ and $y$ proportionality parameters. A slight perspective distortion is observed, thus the average scales are determined by taking into account a major part of the frame. A farther correction is performed in the post analysis after the velocity field is obtained. When required the velocity field is corrected based on a linear approximation of the observed distortions.

3.4. **Hot-Wire Anemometry (HWA)**

3.4.1. **General Scheme**

The system consists of an AN2000 computerized anemometer from A.A. Lab Systems and a carbon film wire probe (70 µm in diameter, about 1mm in length, heavily coated for use in water) from Dantec. The measurement mode is CTA (constant temperature anemometry) where the temperature of the wire is raised above the ambient temperature. The resistance of the active wire is kept constant by running a compensating electric current that dissipates heat and balances the cooling effect of the flow; thus, since
temperature and resistance relate approximately in a linear law a desired temperature is fixed. The current is recorded in term of a voltage $E$. The values of such signal can be attributed with the velocity perpendicular to the wire based on calibration in a specific setup. The general relation is known as King's law, namely:

$$E^2 = A + Bv^{0.45}$$  \hspace{1cm} (3.3)

where $v$ is the velocity, that in principle should be corrected with a small contribution from tangential flow: $v_{\text{eff}} = \sqrt{v_0^2 + \kappa^2 v_t^2}$ \hspace{1cm} (in our probe the yaw coefficient $\kappa$ is 0.2).

The practical accuracy achieved in air can reach 1% [Bruun 1995] but often values of 5-10% are reported [Biswa & al 2002 - Subbarao]. In water there is additional known pitfall that as more the temperature difference increases more micro bubbles of dissolved air are formed near the wire and disturb the flow. Fixing a small temperature difference requires the ambient temperature to be perfectly controlled and the resistance of the wire should be read by a 4-wire method, which is missing in our instrument and probe. Hence the expected accuracy in velocity readings is around 10% in absolute terms. Furthermore, the heavy coating and size of the wire that are designed to withstand the flow impact and electrochemical reaction in water result in a large heat capacitance. Hence, a frequency cutoff appears at 4KHz, much lower than the cutoff in standard air measurements. The cutoff value is determined by testing the duration of an aftershock in response to applied electric pulse to the wire. A care for a visual contact with the probe is required, since occasionally a contamination can be caught on the wire and disturb the results severely.

The point of measurement was chosen in the same plane as the sound detector. When activated simultaneously with sound measurement the radius of position was 12cm. Other positions were at a radius of 8cm and at the center of the cell. The probe was maneuvered by a metal rod of 6mm diameter through a Cajon seal at the wall. The prong of the wire was of the 90 degrees bended type. The wire was aligned perpendicular to the horizontal plane and the readings mostly concern with the velocities in that 2D plane.

**3.4.2. Calibration**

In commercial HWA systems the calibration points are fitted to a curve with 4 parameters at least. I use my own fitting curve determined as:

$$A + BE^2 + CE^4 + DE^6 = v^{0.45}$$  \hspace{1cm} (3.4)
(E should be the true voltage across the wire or at least a single polarity signal).

The flow that is arranged for calibration is produced by co-rotating the plates and placing the wire at a radius of 8cm from the center of rotation. A rigid body rotation is assumed and accordingly the velocity is determined from the rotation speed of the motors. Small bubbles in the fluid decrease the velocity readings, even after most of the dissolved air is eliminated. Hence to assure a proper calibration the condition at each point should be the same. The practice is based on applying a counter rotation flow and then applying the required co-rotation flow during 40 seconds before the measurement and then acquiring during 40 seconds. At rotation speeds of 15-90rpm the waiting time is increased to about 2 minutes.
Chapter 4

Information on a single vortex extracted from sound scattering and compared with PIV

4.1. Method of data analysis

In order to get information about the scattering sound amplitude one needs to find a phase difference between an incident and a scattered signals. Our way to find the scattering wave is simply to measure and subtract the complex sound signals with a flow and at a rest. At high enough frequencies we can concern ourself only with the main change in the wave, which is the phase shift due to change in time of flight by the flow. The phase shift is found from the phase information as $\phi = -\arg\left(\frac{\Psi_{\text{flow}}}{\Psi_{\text{rest}}}\right)$.

The amplitude of scattering wave on the array detector is determined as $A = |\Psi_{\text{flow}} - \Psi_{\text{rest}}|$. Both wave functions, $\Psi_{\text{flow}}$ and $\Psi_{\text{rest}}$ are the central frequency component of a sound extracted from a bank of pulses entered to a Fourier type filter, as described above. Information obtained during a period when the bank of pulses is collected, is called a frame. In a steady flow we average the complex results between available frames. For the dynamic study of a flow we compare the wave function in a flow with the average on many frames to avoid suspecting additional sources of fluctuations. Another step before substitution of $\Psi_{\text{rest}}$ into the last equations is to normalize it by a factor $\exp i\gamma$. The phase correction $\gamma$ is the average in time (between frames) of the values $\gamma'$ that minimize the expression $<|\Psi_{\text{flow}} - \Psi_{\text{rest}}\exp i\gamma'|_y$ (average on $y$ refers to channels of the array). Such step is used to specifically regularize results of scattering from a single vortex and it also compensates for occasionally uncontrolled conditions that change the overall refraction (for example raising of seeding particles by the flow, when they are used). At the end, we obtain the scattering wave function on the transducer array plane as:
\[ \Psi_{\text{scat}} = \Psi_{\text{flow}} - \Psi_{\text{rest}}. \] (4.1)

The presented results of scattered and incident fields are normalized so that the incident field on the emitter plane is of a unit amplitude. To calculate the incident field at a distance \( l \) from its plane we use the Huygens construction (see Eq.(2.18)) that is a propagation transform of the window function (with 5\% apodization at the edges to reduce signal corruption due to noise) \( \Psi(0,y) \):

\[ \Psi(0,y) = \frac{1}{\sqrt{2\pi k_0}} \Psi_{\text{rest}}(0,y') \] (4.2)

Then \( \Psi_{\text{scat}} \) can be calculated from Eq.(2.4).

### 4.2. Characterization of the flow by PIV

We focus on a single vortex, created between co-rotating plates, and find its average properties in time. Since the vortex is axisymmetric we can extract from PIV analysis the profile of an azimuthal velocity versus the radius. The center of the vortex is found according to a minimum in the orthogonal projection of a velocity vector on x and y axes (except for the smallest vortices produced by 25mm rods, where we designated the center by eye recognition). The profile is built by averaging on 64 PIV maps, such as shown in Fig. 4:1a. The profiles for the three inspected flows are shown in Fig. 4:1b (co-rotating 25mm rods), Fig. 4:2 (co-rotating 40mm plates), and Fig. 4:3 (co-rotating 100mm plates).

In general, we see that rotating plates with blades produce a core of a rigid body rotation, i.e., the core velocity increases linearly with the radius. Outside the core, the velocity decreases like in a flow between two cylinders. This solution (Eqs.(2.7)(2.8)) was used to fit the measured velocity profile, and the flow parameters were extracted from the parameters of the fit. Thus we find from the fits \( r_0 = 6\text{mm} \), \( r_0 = 14\text{mm} \) and \( r_0 = 42\text{mm} \), respectively. In all cases generally the core rotation frequency is the same as of the motor.
Fig. 4:1. (a) Velocity field snapshot obtained by PIV for a single vortex created by co-rotating rods of 25mm diameter; (b) Azimuthal velocity as a function of a cell radius extracted from PIV (25mm rods co-rotating at $\Omega=71\text{rad/sec}$) : dots- experimental points, solid line- the model with fitting parameters: the angular speed $\dot{\Omega}=71\text{rad/sec}$ and the core radius $r_0=6\text{mm}$.
Fig. 4:2. Similar to the plot above but for a flow between rotating plates of 40mm diameter and $\Omega=42\text{rad/sec}$. Solid line presents the fitted model with the core radius $r_0=14\text{mm}$.

Fig. 4:3. The same for a flow between rotating plates of 100mm diameter at $\Omega=23\text{rad/sec}$. Solid line presents the fitted model with the core radius $r_0=42\text{mm}$.
4.3. Velocity and circulation measurements via phase shift.

As follows from Eq.(2.15) the measurement of the phase shift provides direct information about the circulation. This result can be obtained from the data in Fig. 4:4, where the spatial dependence of the phase shift obtained from the sound and PIV (here the velocity field measurements were converted into the phase shift via Eq.(2.10)) are presented. It follows that for the maximum phase difference measured in Fig. 4:4 of about $\Delta \phi = 0.7\text{rad}$, the maximum circulation is about $\Gamma = 45,000\text{mm}^2/\text{sec}$. As one can find from
Fig. 4:4 the value of the maximum phase difference and correspondingly the maximum circulation coincide with those found from the PIV measurements within 1-2%.

The phase shift patterns due to the flow based on averaging of 10 frames of 512 pulses for various emitter sizes were examined. It appears remarkably for two types of flow considered (both a single vortex flow with a rigid core rotation, produced by 100mm plates and 25mm rods) that the slope of the phase shift around the center of the beam versus scattering angle (or a distance along the detector) is constant. We describe in detail the flow produced between two co-rotating 100mm plates, which provides stable phase shift plots. In Fig. 4:5 we demonstrate how a value of the phase shift slope $\frac{\partial \delta}{\partial \theta}$ are obtained: the limit is chosen to give a constant slope with the smallest linear regression error. According to Eq.(2.17) the phase shift slope provides direct information about the vorticity of the vortex, $\bar{\omega} = 2\Omega$. The results for emitters of different sizes and at various frequencies are shown in Fig. 4:6a-d. The phase shift slopes for all frequencies and all emitters appear to be linearly dependent on the rotation speed, $\Omega$. The data presented can be either scaled by $\Omega$ or presented via the derivative on $\Omega$, $m = \frac{\partial^2 (\delta / \theta)}{\partial \Omega^2}$. Then the data on

![Phase shift graph](image)

Fig. 4:5. Phase shift as a function of the scattering angle. The solid line is the fit, by which the phase shift slope is defined. The data are for a flow between the plates of 100mm diameter and $\Omega=23$rad/sec with an emitter of 35mm length at $f=4$MHz.
Fig. 4:6. (a) Phase shift slope scaled by sound frequency, \((\partial \phi / \partial \theta)(1/f)\), as a function of angular velocity for a flow between the plates of 100mm diameter and with emitter of 35mm length; (b,c,d) Phase shift slope \(\partial \phi / \partial \theta\) as a function of angular velocity for a flow between the plates of 100mm diameter and with emitters of the 143mm length, 10mm length, and 3.5mm length, respectively, at various frequencies: ■-5.5MHz, ♦-4.5MHz, dots-4.0MHz, □-3.5MHz,▲-2.5MHz, hexagons-2.0MHz, stars-1.5MHz, ▼-1.0MHz, ►-0.5MHz.
the proportionality coefficient \( m \) as a function of frequency, calculated from the plots presented in Fig. 4.6a-d, are summarized in Fig. 4.7. The proportionality coefficient, \( m \), depends linearly on the frequency in the range between 0.5 and 5.5MHz, and the results are sharply separated in two groups: (i) small emitters (3.5, 10mm), and (ii) large emitters (35, 145mm) compared with the size of the detector array of \( d=62 \text{mm} \). The rotation speed of the plates is used to define the flow parameter and is converted to the angular speed of the vortex core obtained from the fits of the PIV data.

According to Eq.(2.17) in the theoretical section, we expect a point-like emitter to have the frequency dependence of the proportionality coefficient, \( K = \varepsilon m / \varepsilon f \), on the plots of Fig. 4.7 as follows:

\[
    K = \frac{\partial^2 (\partial \phi / \partial \theta)_{\theta=0}}{\partial \Omega \partial f} = \frac{4 \pi}{c^2} \frac{r_c^2 r_0}{r_c + r_0} = 0.0281 \text{(sec)(MHz)}^{-1}
\]

The experimental value of \( K \) for 3.5mm and 10mm emitters is \( K = 0.026 \pm 0.001 \text{sec(MHz)}^{-1} \). The value of \( K \) from the plot in Fig. 4.7 for 35mm and 145mm emitters is \( 0.045 \pm 0.002 \text{sec(MHz)}^{-1} \). As explained in section 2.2 the theoretically predicted value of \( K \) for a small emitter should be twice smaller than for a large one, i.e. the theoretically expected value for the large emitters is \( K = 0.056 \text{sec(MHz)}^{-1} \) according to Eq.(2.14). However, specifically for these measurements the large emitters blocked 12% of the total diameter of the exterior of the flow. Assuming effective reduction in the cell radius \( r_c \) the expected theoretical value should be corrected down to \( K = 0.048 \text{sec(MHz)}^{-1} \), and it becomes rather well comparable with the experimental value presented above.
4.4. Sound scattering data.

Using Eq.(2.4) derived in the theoretical section we were able to calculate the scattered signal amplitude from a given velocity field assuming that the emitter wavefront is known exactly. We used a velocity profile of a flow produced by 100mm rotating plates extracted from PIV (Fig. 4:3). The emitter of 35mm length was used and modeled for a flat amplitude (a plane wave) as the input, to obtain the result in Fig. 4:8a. The measured incident and scattered wave amplitudes at the frequency 4.0MHz are compared to the calculated ones (they are scaled to unity value of the wave amplitude at the emitter exit)(see Fig. 4:8a,b). In the calculation of the sound scattering field, a mesh of a half wavelength resolution was used in a zone limited to twice of the emitter size and the velocity field was interpolated based on the velocity profiles values. The incident wavefront was calculated at various points in the cell using Eq.(4.2). The measurement was performed at 5.5Mhz collecting 20×512 pulses in a flow produced by the 100mm disks at the rotation speed \( \Omega = 22 \text{ rad/sec} \). In Fig. 4:9a,b similar data of the incident and scattering sound fields from 125mm emitter at the frequency 5.5MHz are shown (100mm
disks, $\Omega=35$ rad/sec). As in the previous case an agreement with the theoretical calculations is rather good. An evidence of a diffraction pattern (side-lobe) from the emitter of 3.5mm length at the frequency of 5.5MHz in still water (regarded as the incident wave), and in the scattering signal due to vortex flow between the rods of 25mm diameter at $\Omega=71$ rad/sec, is clearly seen in Fig. 4:10a,b. The direction of the initial sound beam was slightly tilted relatively to the direction to the cell center, so that the center of the beam is shifted from the center of the flow. This fact was not regarded in the fitted curve that results in a discrepancy of the measured and calculated sound amplitudes. This result clearly demonstrates that the scattering signal can be easily buried in the incident signal coming from the side-lobes at sufficiently large angles of detection.

Fig. 4:8. Modulus of incident(a) and scattering(b) sound field spatial distributions obtained by the detector array (black dots). Solid lines are the calculations. The flow is produced between co-rotating plates of 100mm diameter, the emitter of 35mm length is used at 4MHz frequency.
Fig. 4.9. Modulus of incident and scattering sound fields, the same as above, but for the emitter of 125mm length at 5.5MHz.

Fig. 4.10. Evidence of side-lobes (diffraction pattern) obtained with emitter of 3.5mm length at 5.5MHz: (a) incident, and (b) scattering sound amplitudes.
4.5. Far field construction of sound scattering field and its relation to velocity and vorticity of the scattering region.

We use the Huygens projection of the near-field scattering signal detected at the receiver array plane, into a synthetic far-field plane using the Rayleigh-Sommerfeld integral via Eq.(2.19). We choose the far-field plane at a distance of \( r_f = 250 \text{m} \), much larger than \( k_v d^2 / 8\pi = 3 \text{m} \). The signal is tapered 2mm on each edge before the projection to suppress numerical instability of the diffraction pattern. By this procedure one obtains full information of amplitude and phase of the scattered signal in the far-field. However, practically due to finite spatial resolution the phase information becomes corrupted. So the next step in the far-field construction is to replace the phase variation as a function of an angle (or a wave number) by a function toggled at each minimum point by a \( \pm \pi / 2 \). In such way we reconstructed the phase field using an observation made in the simulations that at every minimum point of the correct amplitude curve the sign of the field should be inverted. In order to avoid some spurious minimum points the number of digitizing points in integration was increased, and the result was compared against finer digitization mesh. According to Eq.(2.5) the sound scattering field in the far-field is proportional to the Fourier transform \( F_{k_x} \{ v_x \Psi_{\text{rest}} \} \), i.e. in such a way plots of the modulus of the two-dimensional Fourier transform, \( |F_{k_x} \{ v_x \Psi_{\text{rest}} \}| \), in the entire cell were obtained in Fig. 4:11 and Fig. 4:12. The data projected into the far-field are compared with calculations of the scattering field based on the velocity field measurements by PIV. The results are presented in a view angle observed from the center of the cell through the receiver as an aperture. This angle, \( \theta \), is related to the scattering wave vector, \( k_s \), via formulas:

\[
k_{s_x} = k_0 (\cos \theta - 1), \quad k_{s_y} = k_0 \sin \theta, \quad \text{and} \quad k_s = 2k_0 \sin \theta / 2.
\]
Fig. 4:11. Modulus of the structure function of the velocity and the beam function for a flow between the plates of 40 mm diameter and with the emitter of 125mm length. Dots are the experimental data from sound scattering, solid line is calculation based on PIV measurements with the flow parameters $\Omega=42$ rad/sec and $r_0=14$ mm obtained from the fit, and 60 mm characteristic beam size is assumed due to the aperture.

Fig. 4:12. The same as above but for the emitter of 35mm length and the flow parameters $\Omega=23$ rad/sec and $r_0=42$ mm obtained from the fit.
Fig. 4:13. Angular location of the structure function peak, $2\sin(\theta_p/2)$, as a function of the reduced sound wave length $(k_d)^{-1}$.

Fig. 4:14. Value of the scaled structure function, $|F\{v_x \hat{\Psi}\}| \, |g(r_0/d)| \, d^{-0.8}(145/r_c)^{0.2}$, peak, P, as a function of the circulation, $\Gamma$: stars- vortex flow with $r_0=6$ mm created between two 25 mm rods, 60 mm beam width (emitter T4); dots- vortex flow with $r_0=42$ mm created between two 100 mm plates, 60 mm beam width (emitter T4); squares- vortex flow with $r_0=14$ mm created between two 40 mm plates, 60 mm beam width (emitter T4); left-triangles- vortex flow with $r_0=42$ mm created between two 100 mm plates, 35 mm beam width (emitter T2).
We studied properties of the modulus of the Fourier transform, \( |F_{k_z} \{v_x \Psi_{rest}\}| \), at different sound frequencies and different rotation speeds. It was revealed that the angular location of the main peaks in \( |F_{k_z} \{v_x \Psi_{rest}\}| \) is proportional to the sound wavelength (Fig. 4:13). The experimental value of the slope of the plot is 
\[ 2 \sin(\theta_p/2)(k_d d) \equiv k_d d = (1.51 \pm 0.11) \pi \], and is found to be in a good agreement with the value of the slope obtained from numerical simulations \( k_d d = 1.52 \pi \) (see discussion in Sec.2.1). It was also found that the scaled value of the peak height \( P = H d^{-0.8} g(r_0/d)(145/r_0)^{0.2} \) of \( |F_{k_z} \{v_x \Psi_{rest}\}| \) is proportional to the circulation \( \Gamma \) (Fig. 4:14). Thus, four different sets of the data for three different vortices and two beam sizes can be scaled down using the functional dependencies on the beam width, \( d^{0.8} \), and the reduced vortex core size, \( g(r_0/d) \), taken from our numerical calculations (see discussion in Sec.2.1). As seen from the plot scaling does not work great for one set of the data, possibly, due to flow imperfection. On the other hand, the proportionality of the scaled peak height to \( \Gamma \) is not so obvious as one can decide from the first sight, since the finite beam width described by the beam function \( \Psi_{rest} \), alters significantly the function presented in Eq.(2.9). We also found from the measurements that the peak height, \( H \), is independent of the sound frequency and depends strongly (about \( d^2 \) when \( d < 2r_0 \)) on the beam width. When \( d \gg 2r_0 \) the peak height value, \( H \), is proportional to \( r_0^2 \) in a full agreement with the simulations. It is obvious that the finite size of either a beam (an emitter) or a detector (whatever the smallest) limits our knowledge about the scattering field and, therefore, also about the Fourier transform of the velocity field at the wave numbers (or angles) smaller than that corresponding to the peak location.

The projected scattering to a far-field, \( F_{k_z} \{v_x \Psi_{rest}\} \), does not have a direct relation to the structure function of a vorticity, \( F_{k_z} \{(\nabla \times \tilde{v})_z\} \). In order to get the latter, one should extract information about a 2D Fourier transform of the velocity field \( v_x \) from the measurements of \( F_{k_z} \{v_x \Psi_{rest}\} \). However, this transformation is a singular one, so a choice of constrains is made roughly related to the continuity and irrotational character of the flow outside of the acoustic beam extent, where \( \Psi_{rest} \) decays to zero. Particularly for
a plane beam the problem simplifies, since the causing convolution term \( F_{k_x} \{ \tilde{\Psi}_{rest} \} \) can be reduced to a 1D Fourier transform of a window function (y dependent). Now we consider the backward and forward 1D Fourier Transformation on a set of values of \( F_{k_x} \{ v_x \tilde{\Psi}_{rest} \} \) in equally spaced points of \( k_x \), that span about 90 deg. We check that the backward and forward operations cancel each other unless we wish to filter out the identified nuisance of \( \tilde{\Psi}_{rest} \). Indeed, if we regard the result of 1D backward Fourier transformation \( \hat{v}_x(y) = 2\pi \int dk_x y \exp(ik_x y) F_{k_x} \{ v_x \tilde{\Psi}_{rest} \} \) and evaluate it as approximately the one dimensional projection integral of velocity \( v_x^{1D}(y) \tilde{\Psi}_{rest} \) then we can identify that the values decay outside the beam extent \( y = \pm d / 2 \). Next we replace the decayed values by an extrapolation based on the flow model of a single vortex (see Fig. 4:15). The phase in the extrapolated region of \( \hat{v}_x(y) \) should be adjusted having continuity at \( y = \pm d / 2 \) and a \( \pi \) phase difference at the tails. Such modification of \( \hat{v}_x(y) \) provides the filtering step and thus the required field is obtained by completing the forward 1D Fourier transformation, i.e. \( F_{k_x} \{ v_x \} = \frac{1}{(2\pi)^{1/2}} \int dy \exp(-ik_x y) \hat{v}_x(y) \). Now using Eq.(2.6) one can relate in such a way the obtained \( F_{k_x} \{ v_x \} \) to the required structure function of the vorticity \( F_{k_x} \{ (\nabla \times \vec{v})_y \} \) (see Fig. 4:16a).

With a help of the Wiener-Khinchin theorem one can also obtain an azimuthally averaged point by point correlation function of the vorticity, which in the case of an isotropic flow or axial symmetry gives

\[
C_v(r) = \frac{k_x^2}{\pi} \int dk_x J_0(k_x r) \left| F_{k_x} \{ v_x \} \right|^2 \tan^2(\theta / 2)
\]  

(4.3)

The corresponding correlation function of the vorticity is shown in Fig. 4:16b. Since the weight of errors in estimation of \( F_{k_x} \{ v_x \} \) is more pronounced at high scattering angles, the integral in Eq.(4.3) should be cut in a tight range. We used a Gaussian attenuation factor (with \( k_x = 0.7 \text{mm}^{-1} \) characteristic value) on the far-field scattering values to filter out high frequency fluctuations.

In summary, four steps are required in order to obtain the Fourier structure function of vorticity in a single vortex: 1) Construct the reduced far field scattering \( F_{k_x} \{ v_x \tilde{\Psi}_{rest} \} \),
where the argument values are determined based on the minimum peaks in the modulus values. 2) perform a backward 1D Fourier transform. 3) extrapolate the information outside the beam. 4) perform a 1D forward Fourier transform, and apply the relation between the Fourier structure functions of velocity and vorticity. A limiting factor is the noise that dominates above a certain $k_s$ value.

Fig. 4:15. Reconstructed one-dimensional velocity field $(v_x)_D$ according to the procedure described in the text (for 60 mm beam width, for a flow between two plates of 40 mm diameter and $\Omega=42$ rad/sec, and an emitter T4 at frequency 5.5 MHz). Dashed line is an extrapolation based on the flow model.

It is obvious that the vorticity structure function restored in such way contained additional information reflecting our guess (or our PIV results). However, this additional information is mostly relevant to a low wave number range of the vorticity structure function, i.e. to the core of the Fourier transform of the vorticity. At wave numbers larger than the core wave number value, the vorticity structure function provides information based on the original experimental data. Anyway, the entire procedure is applicable exclusively in the single vortex case. In analysis of turbulence we eventually do not filter out $\tilde{\Psi}_{\text{cov}}$ but rely on the randomness of the flow and the statistics in time to smooth and cancel small diffraction oscillations caused by diffraction from the beam and aperture.
Fig. 4:16. (a) Modulus of the Fourier transform of the vorticity obtained from the data in Fig.4:15 (b) Normalized spatial correlation function of the vorticity. Dashed line-sound scattering results, solid line-calculations based on PIV velocity profile measurements as Fig.4:2 (with extrapolation up to the cell wall) with $r_0=14$mm and $\Omega=42$rad/sec.
4.6. Dynamics of a vortex measured by sound scattering.

The sound scattering technique can be used to study vortex dynamics. It can be studied by a phase dynamics approach to get temporal variation of a vortex location, vortex radius, and vortex circulation similar to what was done in [Manneville & al 1999,2001]. However, we used a different approach of sound scattering and compare the results on a vortex precession with those obtained by PIV. The vortex position is found from the minimum point of the scattering amplitude pattern. The periodicity in the vortex motion is characterized by the variation of the peak heights in the far-field scattering pattern. The setup is used with the upper plate of 280mm in diameter at the angular speed $\Omega$=30 rad/sec, and the lower plate is absent. The rate of PIV tracking is one map per 0.266 seconds. The rate of ultrasound tracking based on collection of 32 pulses per frame is one wave function plot per 0.016 seconds (sound frequency is 2.5MHz, emitter is 96mm long). In the case of the rotating upper plate, a periodic precession of the vortex is found by both techniques: PIV and the sound scattering (see Fig. 4:17). To compare two sets of the data the time correlation functions of the vortex locations were produced from both sets. A good quantitative agreement between the dynamic results of PIV and ultrasound measurements was found (see Fig. 4:18) with a period of 0.90 sec. Besides, the peak height of the Fourier transform of the velocity projected into a far-field is found to be twice periodic compared with the vortex position (see Fig. 4:19a). It is clearly seen in the auto-correlation function presentation of the data in Fig. 4:19b. This effect occurs due to increase or decrease in the integral of $v_x$ over the beam area (detector view), when the vortex core is shifted sideways, since for small $k_x$ $\left| {\mathcal{F}}_{k_x} \{v_x \Psi_{rec}\} \right|$ contains a contribution from the integral of $v_x$, and double periodicity shows up due to absolute value of the function. The precession frequency as a function of the rotation frequency obtained from the auto-correlation functions similar to that shown in Fig. 4:19b, is presented in Fig. 4:20. We were not able to get such features from the PIV measurements due to restrictions in accuracy; however, the ratio of 1/5 between the precession frequency to the rotation speed of one plate is not far from expected according to HWA measurements shown in [Labbe & al 1995 Fig.5].
Fig. 4:17. Temporal dependence of the vortex location detected by PIV (dots in the upper set) and by sound scattering (squares in the lower set) measurements.

Fig. 4:18. Auto-correlation functions for two sets of the data in Fig.4:17 as a function of the time delay. Solid line- for sound scattering, dash-dotted line-for PIV.
Fig. 4:19. (a) Temporal dependence of the peak of the modulus of the velocity structure function for the same measurements as in Fig.4:17. (b) Auto-correlation function for the data in (a) shows that the vortex precession frequency is half of the peak frequency.

Fig. 4:20. Vortex precession frequency obtained from the auto-correlation functions of the far-field scattering peak time dependence (similar to the plot in Fig.4:19b), as a function of the frequency of the plate rotation. The proportionality factor is 0.22±0.03.
4.7. Conclusions.

We demonstrated that our system provides reliable information about both the phase and the amplitude of the sound scattering signal by acquiring simultaneously 62 channels of the detector array. The spatial and temporal information on the phase of the scattered signal allows us to get values of circulation, vorticity, vortex location, and vortex core radius. We verified quantitatively the theoretical value for the slope of the proportionality coefficient, \( K \), for a single rigid body rotation vortex in a finite size cell. This method is rather comparable with the acoustic time-reversal mirror (TRM) method [Manneville \& al 1999-2001]. Instead of scattering amplification due to number of crossing of the flow in the TRM method, in our method the signal-to-noise ratio is amplified by averaging complex wave functions over many pulses. At the same time in our approach we can also use the amplitude of the scattering signal to characterize the flow. We have shown that it is possibility to obtain reliable information about velocity and vorticity fields of a single either stationary or time-dependent vortex flow by the acoustic scattering technique with a finite width sound beam of the order of the vortex size and a finite size receiver taking a scattered signal rather close to the scattering region. The scattering results suit the information obtained from the PIV measurements taken simultaneously on the same vortex. The achievement is also due to comparing the experimental results with a specific analytical solution for the far field scattering in the case of a rigid body flow inside a closed compartment. From the theoretical side we realize that the only empirically relevant calculation of scattering is that of a confined flow, since an emitter and a detector define the flow perimeter, hence the importance of the analytical solution. From the experimental side, we say that using a multi-channel acquisition system is obligatory, if one wishes to analyze near field scattering data.
Chapter 5

Measurement of the Fourier structure function of vorticity in turbulence

5.1. The approach of sound scattering in turbulence

The finite size of the sound beam remains an obstacle in revealing essentially low $k_s$ values of the Fourier structure function of vorticity, if mean flow is considered. However, this is not the case in turbulence, since we study the structure of fluctuations, of which the significant scales are much smaller than the size of the beam (or the system size). Fluctuations are mostly uncorrelated over the system size. The procedure specified in the analysis of stationary flow can be skipped; the far field construction is the only process applied in turbulence analysis. This task is achieved due to the simultaneous acquisition of sound pulses on many acoustic detectors, on which the Huygens construction can be applied. The duration of sound propagation through the cell is a typical freezing time segment (200µs). Within this period one pulse is sent, and the flow features are drifted less than the size of the acoustic elements (1mm). In looking on the fluctuation of a wavefunction captured by a single pulse it is possible to construct the far field pattern and to obtain the fluctuation of the Fourier structure function of velocity $\delta | F_{k_s} \{ v_s \} |$, or vorticity $\delta | F_{k_s} \{ (\nabla \times \vec{v})_s \} | = 2 k_0 \tan(\theta / 2) \delta | F_{k_s} \{ v_s \} |$. The final step in analysis is to average these fluctuations in absolute values over the many sound pulses.

The scattering information, if mapped on the 2D Fourier domain, unravels only a line of values of the structure function of vorticity (or vorticity fluctuation). The line is demonstrated in Fig. 5:1, along with other possible results (in broken lines) that would be acquired if at the same time additional sound beams are emitted in many directions and measured. In principle, sending sound beams in multiple directions and arranging appropriate detector arrays allows retrieving the complete function of vorticity, thus vorticity may be reconstructed. Since such experiment is beyond our efforts we must count on the isotropy of the turbulent flow, and expect a symmetry in relation to $| k_s |$.  

60
5.2. Direct observation of the energy spectrum $E(k)$

Upon assuming isotropy and homogeneity of the turbulent flow the scattering method provides rather a direct observation of the energy spectrum in the spatial domain. I define the kinetic energy spectrum per unit mass by a representative sample plane via:

$$
\int dkE(k) = 3 \int \frac{d^2r}{A} \frac{v^2}{2},
$$

where $A$ is the sampled area, in this case the intersection of the beam with the flow. We neglect the variation of the velocity in the perpendicular direction over the thickness of the beam of 1cm. Due to a property of Fourier transforms (see appendix.A.2) $E(k)$ is retrieved at $k = |\vec{k}_s|$ from the Fourier structure function of vorticity via:

$$
E(|k_s|) = \frac{6\pi^3}{A|k_s|} < |F_{k_s} \{(\nabla \times \vec{v})^2\}|^2 >
$$

(5.1)
The results are shown below for an average of 60,000 pulses at 1.8KHz repetition rate. In order to demonstrate the role of far field construction we present in Fig. 5:2a a typical pattern of |\( \delta F_s \{v_s\} \rangle \) extracted by our method, versus the inset in Fig. 5:2a that shows the same result if one pretends that the scattering field directly on the transducer is approximately a far field. Indeed, the former is a pattern that was shaped by the effects of turbulence and the far field propagator, whereas the latter is roughly a flat curve of the average intensity of fluctuation. There is not a slight resemblance between the figures (the data is based on counter rotation flow at Re = \(1.5 \cdot 10^6\) with 3MHz sound).

In Fig. 5:2b the energy spectrum plot is shown based on the data in Fig. 5:2a and based on Eq.(5.1). The dotted line denotes the -5/3 slope according to the Kolmogorov law; indeed, in some range the spectrum follows the expected law. However, some irregularity appears outside the values of \(0.1 < k < 1 \text{mm}^{-1}\), where the characteristics of the flow cannot be retrieved. The lower side of the range in k values concerns with the finite size of the beam that plays a limiting factor. On the other side, the higher k values correspond to exceeding the visibility of large scattering angles through the detector aperture (i.e some sound rays are blocked by the limited length of the detector array). According to the size of detector array and the cell diameter, at angles larger than 6° the visibility starts to deteriorate (see Fig.5:2c). The corresponding limit is \( k_{S_{\text{max}}} = 2k_0 \sin(\theta_{\text{max}}/2) \approx 0.1k_0 \) (about 1mm\(^{-1}\) at 3MHz). We checked the results at various frequencies as presented in Fig. 5:3. Indeed the aperture finite size matters less when increasing the sound frequency (or wavenumber \(k_0\)). Our instruments are unable to function properly at 7MHz and above. Still, in general, at (higher) k values that exceed the periodicity of the detector array (its resolution) a farther limitation emerges. A synthetic propagation via the Fresnel-Huygens construction cannot provide new information on the gradients of velocity that is not contained, by some representation, in the level of details in which the initial field is acquired. Hence, the limiting factors in availability to extract the characteristic energy spectrum, \(E(k)\), are: 1) the size of detector array, 2) the number of elements, and 3) the highest sound frequency that can be detected with good signal to noise ratio.
Fig. 5:2. Results extracted from sound scattering in turbulence (at 3MHz frequency, counter rotation at Re=1.5·10^6). (a) Average fluctuation of velocity structure function attained by far field construction; the inset – a false result by removing the far field construction process. (b) The energy spectrum derived from a. (c) Scheme of the aperture limit for scattering element near the emitter.
Fig. 5:3. The effect of sound frequency variation due to visibility limitation by the detector aperture. The correct energy spectrum plot is apparent from the common coalescence of the curves (the most consistent is the 7MHz curve). The results are obtained in counter rotation flow at 5rev/sec.

Useful information in terms of the dissipation of a turbulent flow, $\varepsilon$, can be gained from combined scattering measurements at several Reynolds numbers. There is a known scaling law of the energy spectra [Saddoughi & al 1994; Nelkin 1992; Gotoh 2001] that appears on plotting $\varepsilon^{-2/3} \eta^{-5/3} E(k)$ versus $k \eta$, where $\eta$ is the Kolmogorov scale defined as $\eta = (\nu^3 / \varepsilon)^{1/4}$ (refer to appendix.C). The idea is to find the best match between the scaled spectrum lines by fitting values of $\varepsilon$. It appears that the scaling exist in our results at all rotation speeds (except for a few cases at low rotation speeds that did not converge). Examples of our results are shown in Fig. 5:4. The dependence of the dissipation, $\varepsilon$, on the Reynolds is presented in sec.6.3 where a missing factor is determined from Eq.(3.2). The dependence is found as $\varepsilon \sim Re^{3.2 \pm 0.1}$ with good quality of data, the exponent is near the expected result $=3$ according to dimensional analysis [Cadot & al 1997].
The constant $C$, the one of an order of unity in Kolmogorov equation, can be determined experimentally. In a scaled axes the equation is $\varepsilon^{-2/3} \eta^{-5/3} E(k) = C(k \eta)^{-5/3}$, and the fitted number is found $C \approx 0.8$ (based on Fig. 5:4). A comparison with HWA spectra is available and is described in the next chapter.

Fig. 5:4. Scaling of the energy spectrum in counter rotation flow at different speeds (Re numbers), based on sound scattering, at the following planes: (a) $h=-30\text{mm}$, (b) $h=0$. 
Chapter 6

Study of time domain spectra by HWA and sound phase shift

6.1. Energy spectrum acquired by HWA

In order to utilize the results from hot-wire anemometry one needs to adopt both the assumptions of isotropy/homogeneity and the Taylor hypothesis. In addition, when the probe is placed at several points of different radii in the cell, the results are expected to be similar due to homogeneity. I define the time series spectrum, $E(f)$, of the kinetic energy of flow in three coordinates, via:

$$
\int dE(f) = \frac{3}{2} \frac{\|v\|^2}{T} (v_\perp \text{ is the velocity vector projected on the 2D plane perpendicular to the hot-wire; } T \text{ is the sampling time}).
$$

Thus the energy spectrum (with $f>0$) is determined by:

$$
E(f) = \frac{3}{2T} <\int_0^T dt \exp(-i2\pi ft)v_\perp|^2>
$$

(6.1)

The average sign in Eq.(6.1) indicates that we average on an ensemble of cases (required in practice due to shifts in the response of the HWA bridge). A spectrum result is derived by collecting about 200 segments of time into the average, which acts in smoothing the spectrum plot. We produce each plot based on $4 \cdot 10^6$ points in total that are sampled at a frequency of 8KHz and low passed at 2KHZ using a high order Chebyshev filter.

In Fig. 6:1 we compare a turbulent flow and a flow of a single vortex attained by reversing the direction of one plate. The spectrum in the counter rotation mode ($Re=1.5\cdot10^6$) reveals a Kolmogorov law between frequencies of 100Hz-1KHz. In the co-rotation mode, where turbulence is undeveloped, sharp peaks emerge especially at the frequency of rotation, 6Hz, and at 4 times larger frequency due to the 4 blades on each plate.
Knowledge of the average advection velocity in the turbulent flow is a required step in analysis of the spectra. We present in Fig. 6.2 the mean and the STD statistics of the velocity readings by the HWA probe at various Reynolds numbers and positions. In most cases we see that $|v_\perp|$ and $STD(|v_\perp|)$ are proportional to the Reynolds number (to the speed of rotation). The errors increase at higher speeds due to increased dissipation rates and a small uncontrolled temperature drift. The velocities near the boundary (25mm from the wall of the cell) are reduced by 40% in the mid-plane measurements in relation to the rest of the points inside the cell (in the same plane). In summary, the values at the mid plane are $<|v_\perp|> = 1.4 \cdot 10^{-6} \text{m/s Re}$ and about 45% of it is the STD values (not the same as fluctuations level since the flow is not unidirectional). In the shifted plane, moved by 30mm towards the lower plate, the readings are: $<|v_\perp|> = 0.9 \cdot 10^{-6} \text{m/s Re}$, $v'_{rms} \approx STD(|v_\perp|) = 0.50 \cdot 10^{-6} \text{m/s Re}$. The fluctuation level is about 55% of the advection velocity in the $h=\pm 30$mm layer. Since the fluctuation level is found uniform in counter
Fig. 6.2. HWA advection velocities in counter rotation flow at different Reynolds and positions. (a) average velocity in the mid-plane, (b) average velocity in a shifted plane $h=-30$mm, (c) STD of velocity in the shifted plane. Full circle (●) and plus(+) are two sets of measurements at $r=80$mm (with different direction of the prong of the wire), diamond (◊) – at $r=0$, empty circle(○) – at $r=120$mm (close to the wall of the cell).
rotation flows, the fluctuation level is usually compared with the largest velocity near the plates $\Omega R$. Our result gives $v'_{rms} = 0.14\Omega R$. For comparison, in [Dernoncourt et al 1998] the result for counter rotation flow in water with plates of 99mm radius of 100mm apart gives $v'_{rms} = 0.12\Omega R$. In [Zocchi & al 1994] the result in low temperature Helium gas is $v'_{rms} = 0.17\Omega R$ measured 2cm from a plate of 100mm radius. So the ratio is universal.

In order to present the energy spectra $E(k)$ a conversion formula is utilized as:

$$ 2/\left|v\right|_{\perp} \pi = \langle \vec{v}_{\perp} \rangle, $$

where $\left|\vec{v}_{\perp}\right|$ is the velocity sensed by the hot-wire (nearly the advection velocity). From the conversion we obviously write $E(k) = E(f) \left|\vec{v}_{\perp}\right| / 2\pi$. Scaled plots of $\varepsilon^{-2/3} \eta^{-5/3} E(k)$ versus $k \eta$ are shown in Fig. 6:3 for different cases in counter rotation flow. Now we compare the energy spectrum with the results of sound scattering measurements (indicated in a dashed line), which also mark the inertial regime according to the Kolmogorov law. The fitted values of $\varepsilon$ are shown in section 6.3.

![Energy spectrum from HWA. The probe is located at the shifted plane $h=-30mm$ (at $r=80mm$).](image)
Fig. 6:3b. Energy spectrum from HWA. The probe is located at the mid-plane $h=0$ near the boundary (at $r=120\text{mm}$, the influence of the blades is apparent in the scattered peak).

Fig. 6:4. Another representation of the results in Fig.6:3. The maximum location reveals the ratio between Taylor's scale and Kolmogorov's scale. (a) $h=-30\text{mm}$, (b) $h=0$.

In Figs.6:4a-b that show a scaling of $\varepsilon^{-2/3} \eta^{-5/3} E(k)(\eta k)^2$ vs. $\eta k$ the peak location $(k\eta)_{\text{peak}}$ can be interpreted (based on Eq.(8.8) in appendix C) according to the following approximate formula:
\[ \lambda_T = \frac{\sqrt{5}}{k_{\text{peak}}} = \frac{\sqrt{5}}{(k\eta)_{\text{peak}}} \eta \] (6.3)

Namely, we can estimate the ratio between the Taylor scale and Kolmogorov’s scale based on the scaling in the energy spectrum. The results of the peak locations for the shifted plane and the midplane spectra are \((k\eta)_{\text{peak}} = 0.07\) and \(0.03\) respectively. Thus the microscales ratios are \(\lambda_T / \eta = 32\pm5\) and \(75\pm10\), respectively.

In appendix C, specifically Eq.(9.4) and Eq.(9.9), we discuss a method to estimate the micro-scales based on the dissipation rate and the fluctuation rms. Our estimate of the dissipation rate in counter rotation mode is \(\varepsilon = 4.9 \times 10^{-18} \text{W/kg Re}^3\), as suggested from Eq.(3.2). The fluctuation level is summarized via Fig.6:2c as \(v'_{\text{rms}} = 0.50 \times 10^{-6} \text{m/s Re}\). Thus the calculated ratio between the scales is in the range 30-46 (see Fig.6:5), which is found comparable to the value \(\lambda_T / \eta = 32 \pm 5\) stated based on the spectrum in Fig.6:4a. In a given range of Reynolds numbers, the lines of \(\lambda_T\) and \(\eta\) are approximately parallel, and thus the scaling manifestation of the energy spectrum (leading to Eq.(6.3)) is not rejected. As a note, we add that the values of \(R_\lambda\) can be determined by Eq.(9.10) as \(R_\lambda = 200-570\) for Reynolds between \(2.5 \times 10^5 - 1.7 \times 10^6\) \(\text{Re}^0.5\), based on the measured factors. In Helium gas [Zocchi &al 1994] in the same Re there are different values \(R_\lambda = 560-1460\).

![Fig. 6:5. Estimate of the Taylor micro-scale and the Kolmogorov scale in counter rotation flow.](image)
6.2. The spectrum of sound phase shift fluctuation

In the geometrical acoustic approximation the phase shift of sound is related to an integral of velocity along the path of propagation, \( \phi(y) = (k_0 / c) \int_0^L \! dx' v_x(x', y) \) \((L=290\text{mm})\) is the length of acoustic path). Here we do not require simultaneous multi-channel acquisition but we take it into advantage as providing ready extended samples. Any quantity either the spectrum or a statistical moment is extracted from each detection element separately, and then the collection are averaged. The data of phase shift is represented in terms of a new variable that is introduced:

\[
v_p \equiv \int_0^L dx' v_x / L
\]

We discuss the energy spectrum calculated in similar way to Eq.(6.1), i.e

\[
E(f) \sim \frac{1}{T} \left| \int_0^T dt \exp(-i2\pi ft)v_p \right|^2
\]

The idea behind Eq.(6.5) is that at most of the points along the acoustic path the local velocities exhibit a similar energy spectrum, thus \( |F_f\{v_x\}| \equiv \left| \frac{1}{T} \int_0^T dt \exp(-i2\pi ft)v_x \right| \) is independent of \( x \). We change the order of integration with respect to \( t \) and the acoustic path, and Eq.(6.5) is found proportional to \( \langle F_f\{v_x\} \rangle \). Though, the phases of \( F_f\{v_x\} \) sum up to a non-unity factor. A similar correspondence but fixed in factor was derived by [Tatarski 1961 sec.6.2] for the spatial spectra of phase shift and the index of refraction. In order to resolve the missing factor for the energy spectrum dependence on \( v_p \) we integrate the right-hand side of Eq.(6.5) with respect to the frequency:

\[
\int_0^\infty df \left\{ \frac{1}{T} \left| \int_0^T dt \exp(-i2\pi ft)v_p \right|^2 \right\} = \int_0^T dt \int_{-\infty}^\infty \frac{df}{2\pi} \frac{1}{T} \left| v_p(t) \right|^2 \exp(i2\pi ft) = \frac{1}{2} \left< v_p^2 \right>
\]

The next step requires a definition of the normalized correlation function:

\[
f(x) = \left\langle \frac{[v_x(x'+x) - \bar{v}_x][v_x(x') - \bar{v}_x]}{\left< [v_x(x') - \bar{v}_x]^2 \right>\right.\right.\]
where $\nabla_x \equiv < \nabla_x >$ and the average is assembled by variation of the time and $x'$. This function is related to the integral scale of the flow, $L_t$, as:

$$L_t = \int_0^\infty dx f(x)$$  \hspace{1cm} (6.8)

[Piquet 1999; Tatarski 1961]. $L_t$ relates to the size of the largest eddy.

Additionally, a relation between the phase shift related variable and the velocity is calculated:

$$< [v_p - \nabla_p]^2 > = \frac{1}{L} \int_0^L dx' < (v_x - \nabla_x)(v_x - \nabla_x) > / L = \frac{2}{L} \int_0^L dx f(x) < [v_x - \nabla_x]^2 > , \hspace{1cm} (6.9)$$

assuming $L$ is large, such that $\int_0^L dx'/L \approx \ldots$. (see for example [Beran 1968 chp.6]).

Hence, the variance of $v_p$ when $L \rightarrow \infty$ is found proportional to the variance of velocity:

$$< [v_p - \nabla_p]^2 > = \frac{2L}{L} < [v_x - \nabla_x]^2 >$$  \hspace{1cm} (6.10)

Since by definition $\int_0^\infty d\Omega E(\Omega) = \frac{3}{2} < (v_x - \nabla_x)^2 >$ (for velocity in 3 coordinates; for spectrum with $f>0$) and by Eq.(6.6) and Eq.(6.10) the spectrum of energy is calculated:

$$E(f) = \frac{3}{2} \left( \frac{L}{2L_t} \right) < \int_0^T dt \exp(-i2\pi ft) v_p(t) >$$  \hspace{1cm} (6.11)

There is significant information in the acquired statistical variance of $v_p$, which concerns with the integral scale of the flow. The variance of any velocity component is the same due to isotropy; hence, by Eq.(6.10), a comparison can be made with the variance of $v_{\perp}$ acquired by HWA:

$$< [v_p - \nabla_p]^2 > = \frac{2L}{L} < [v_{\perp} - \nabla_{\perp}]^2 >$$  \hspace{1cm} (6.12)

The proportionality factor can be revealed from parallel measurements of phase shift and HWA, and the integral scale $L_t$ can be obtained.

In Fig. 6:6 the square root of the variance of $v_p$ is presented. About 10 points are plotted for each rotation speed (the scatter of data can be assessed). Each point is based on a sample file collected during 18 seconds from the 62 acoustic channels at pulse
repetition rate of 1.8KHz. The frequencies of sound are 3MHz (for the h=-30mm measurement) and 2.5MHz (for mid-plane h=0 measurement). Type T5 emitter is used. The results show a linear relation of the fluctuation level with the rotation speed. If the frequency is increased to 5.5MHz the $2\pi$ wrap around of the phase shift starts to take place at $Re=10^6$, as indicated by improper values of $\text{STD}(v_p)$ in comparison to lower frequency values (such limitation does not exist in the scattering analysis). The fitted lines in Fig. 6:6 for the h=-30mm and h=0 cases are the same. We determine by comparison to the HWA data based on Eq.(6.12) that the integral scale length is $L_i = 51 \pm 4$ mm.

Fig. 6:6. Fluctuation level of the phase shift (in velocity units) in proportional relation to the Reynolds number. Measured at the planes: (a) h=-30mm, (b) h=0.
In Fig. 6:7 the spectra of kinetic energy are presented based on the spectra of the phase shift fluctuations in time. The raw data is the same as used for above, with average on 10 samples of 18sec, each sample is cut to 20 slices in time for more average of the spectrum (total number of phase shift points per $Re$ number is $2 \cdot 10^7$). The scaling in the plots is obvious (the fitted dissipation rates are shown in the next section). We use the Taylor conversion based on the advection velocities measured by HWA (as described in section 6.1). The comparison with the results of the scattering method (from chapter 5) and the HWA results show some disturbing disagreements, as follows.

The scaled spectrum from scattering data (as Fig. 5:4) is available in the range $0.004 \leq k\eta \leq 0.06$, frankly narrower than the line plotted in Fig.6:7. The phase shift scaled spectrum barely matches the available scattering results. There is no collapse between the phase shift and the HWA plots. One can point out, however, that the comparison in magnitudes is of secondary importance to the points where the slope deviates from a $-5/3$ law. If the Taylor conversion factor is varied, then the selected inertial range (in $k\eta$) is shifted and also the magnitude is shifted. By comparison to HWA spectrum (noting also the line of $Re=1.7 \cdot 10^6$ that bulges out of the common scaling of the phase shift spectra), we still can suspect that the phase shift spectra do not exactly follow the kinetic energy of the flow. Our choice to explain the result is that the geometric approximation of sound deteriorates with increasing of $k$, (inverse to the features size of the flow). This happens in the condition that the wavelength (of 0.6mm) is only marginally smaller than the Taylor scale of the flow (estimated 1.8-0.7mm).
Fig. 6:7. Scaled energy spectrum based on phase shift fluctuations of sound, shown in comparison with the two other methods: sound scattering (indicated by a -5/3 line) and HWA. The conversion from time to space domains is based on the advection velocity measured by HWA. (a) h=-30mm, f=3MHz, (b) h=0, f=2.5MHz.
6.3. Comparison of scaling results

In Fig. 6:8 we show the fitted dissipation rates that result from the scaling of the energy spectra by the different methods. This analysis provides values with an undetermined factor, which is fixed based on an estimate of the dissipation rate derived from the torque measurements, Eq.(3.2) (also crossed with other methods). The estimate line (with power law of 3 in Reynolds) is indicated as a broken line in the plots. The conclusion for the scattering and HWA spectra is that the scaling parameters match the expected ones. The phase shift fitted dissipation rates follow the same line with a bias error of 0.2 Watt/kg.

Fig. 6:8. Comparison between the relative values of the dissipation rate fitted for the scaling of the energy spectra by the three methods discussed above. The line indicates the estimated dissipation rate of power 3 in Re, by Eq.(3.2). The two cases are (a) for the plane at h=-30mm, (b) h=0.
6.4. The spatio-temporal iso-correlation map

The availability of an array of detectors allows us to examine the spatio-temporal symmetry suggested by the Taylor hypothesis. The velocity correlation function that was used for this purpose by Ho and Kovasney [Ho & al 1976] may better be replaced by a function of velocity difference. We plot here the contours of the 2D function \( f(\Delta y, \Delta t) = \langle v_p(y' + \Delta y, t' + \Delta t) - v_p(y', t') \rangle \), which serves as our correlation indicator. \( v_p \) is derived from the phase shift fluctuations (refer Eq.(6.4)).

The results plotted in Fig. 6.9 suggest that the iso-correlation map is the same for different cases of rotation speeds or planes (h=0 or -30mm) in the counter rotation flow. The maps reveal a topology of circles, exactly as expected in the Taylor hypothesis, except that we allow the conversion factor (assumed around \( \langle |v| \rangle \) ) to be approximate in level of an order of magnitude (note that similarly the factor remained unspecified in [Tennekes 1975]). It is a surprising fact that the structure of the contour appears even though the array of detectors is not placed along a stream of flow. The source of symmetry in fluctuation in time and space comes from the complex character of the flow, not from a large eddy. The average velocity vanishes in the case of h=0 and even if a temporary main flow exists, a large eddy should not contribute to the iso-correlation map, since the advection in the y direction (the axis of the array of detectors) summed over the acoustic path (in the x direction) is almost canceled by the symmetry of the eddy. Yet as another point to mention, the correlation does not seem to vanish at distances of several \( L_1 \). At such distances, the transformation between the time and space domains appears to go in nonlinear regime; the proportionality between the crossing points of the contours on the two axes varies with the correlation distance, which indicates a new phenomena not included in the Taylor hypothesis.

The main conclusion from Fig. 6.9 is that the Tennekes (or extended Taylor) hypothesis is principally correct. The exception is that the conversion factor between the time and space domain is about 2-3 times less than the advection velocity measured by HWA, and the factor still decreases at increasing correlation distances.
Fig. 6:9. Iso-correlation maps in counter rotation flow. (a) h=-30mm, 360rpm. (b) h=-30mm, 180rpm. (c) h=0, 360 rpm.
Chapter 7

Moments of velocity difference ("structure functions")

7.1. Hypothesis

Here I introduce a possible interpretation to statistical moments of the phase shift difference in time, and explain their usefulness. A n-order moment of phase shift difference in time is denoted (already in spatial form) as

\[ S_{p,n}(x) = \left\langle [v_p(t') - v_p(t'+x/\|v\|)]^n \right\rangle, \]

where the average is on \( t' \). \( v_p \) is the average velocity component along the acoustic path in the \( x \) direction. The model is strongly based on the Tennekes hypothesis (or extended Taylor hypothesis).

As an example, the second order moment is written explicitly

\[ S_{p,2}(x) = \frac{1}{T} \int dt' \frac{1}{L} \int dx' dx'' [v_x(t',x') - v_x(t' + \frac{x}{\|v\|},x')][v_x(t',x'') - v_x(t' + \frac{x}{\|v\|},x'')] \]

\[ (7.1) \]

Some of the terms are treated as described in sec.6.2, e.g.

\[ \left\langle \frac{1}{L^2} \int dx' dx'' v_x(t',x') v_x(t',x'') \right\rangle = \frac{2L_1}{L} < v_x^2 >, \]

\[ (7.2) \]

where \( L_1 \) is the integral scale. The more problematic term is the spatio-temporal correlation \( <v_x(t'+x/\|v\|,x') v_x(t',x'')> \). There seems to be a condition \( x'' = x' - x \) that has the maximal weight in the correlation; however, this contribution in a three dimensional flow is fast decaying. It rarely happens that the flow advection trajectory meets a single direction \( x \) along considerable time. Thus, the second assumption I introduce is independence between the space and time variables, namely:

\[ <v_x(t_1,x_1) v_x(t_2,x_2)> = f(<v|>(t_1-t_2))f(x_1-x_2) < v_x^2 >, \]

\[ (7.3) \]

where \( f(x) \) is the normalized correlation function as defined in Eq.(6.7). As a result :

\[ \left\langle \frac{1}{L^2} \int dx' dx'' v_x(t' + \frac{x}{\|v\|},x') v_x(t',x'') \right\rangle = \frac{2L_1}{L} f(x) < v_x^2 > \]

\[ (7.4) \]
Hence, summing the terms in Eq.(7.1), we can relate the second order moment of phase shift difference to a the moment of local velocity difference, defined basically as $S_n(x) = \langle [v_x(t') - v_x(t' + x / \langle \|v\| \rangle)]^n \rangle$ (see refined definition below). From Eq.(7.3) we expect $S_2(x) = 2 < v_x^2 > (1 + f(x))$, thus finally:

$$S_{p2}(x) = \frac{2L}{L} S_2(x) \tag{7.5}$$

In the third order moment of $v_p$ we look on the triple integral terms:

$$S_{p3}(x) = (\frac{2L}{L})^3 S_3(x) \tag{7.6}$$

### 7.2. Correction of the Taylor hypothesis

We take a step further in the construction of velocity difference moments $S_n(x)$ from HWA time domain velocity measurements. In the results that we present $x$ is not $<\|v\| \times t \rangle$ (product of the average velocity perpendicular to the wire $v_\perp$ and a delay time $t$), but rather the specific separation between any two points of time $t_1$ and $t_2$:

$$x = \int_{t_1}^{t_2} |v_\perp(t')| dt' \tag{7.7}$$

(In HWA experiment $x$ denotes the instantaneous advection direction, assuming it does not vary during correlation time. Due to isotropy the results should correspond to the direction of the beam in a sound experiment.).

$S_n$ is calculated based on a conditional ensemble

$$S_n(x) = \langle [v_\perp(t_1) - v_\perp(t_2)]^n \rangle \text{ for all cases that } \sum_{t_1 \leq t' \leq t_2} v_\perp(t') = x \tag{7.8}$$
Discretization problems are handled by our software in terms of linear interpolation between adjacent points of $x$, and the algorithm is efficient enough to run on a desktop computer.

The purpose of the correction technique is to treat data of HWA measurements at the middle way between counter rotating plates, where the average velocity $<v_\perp>$ vanishes. The direction of advection still varies in time; however, during the time required to fill a separation of the size of the cell the large scale flow does not vary considerably. We can assume the measurement effectively involves one component of velocity.

### 7.3. Third order moment compared between HWA, sound, and PIV

Our three goals are to compare HWA to phase shift measurements, and secondly to obtain in an absolute way the dissipation values, $\varepsilon$, and third, to present the results in compatible way to theoretical predictions.

Specifically, the results of the third order moments, $S_3$, can be scaled according to the prediction of the Karman-Howarth equation. We quote the corrected equation that includes in the last term a contribution from the pumping energy [Moisy & al 1999]:

$$\frac{-S_3(x)}{x} + \frac{6\nu}{x} \frac{dS_3}{dx}(x) = \frac{4}{5}\varepsilon(1 - \frac{5}{14}\frac{x^2}{L_f^2})$$  \hspace{1cm} (7.9)

$L_f$ denotes the length scale that concerns with the pumping energy according to this approach. In our case the minimum resolved spatial separation, $x$, is much higher than the Kolmogorov scale $\eta$, and then the term with the kinematic viscosity $\nu$ can be neglected. Therefore, we present the measurements data as $-S_3(x)/x$ versus $x/r_c$ ($r_c = L/2$ is the cell radius). Data from phase shift measurement is interpreted via Eq.(7.6), and was based on absolute value of differences in $v_p$ over time.

In Figs. 7:1-7:5 we present results of the third order moments (over separation distance) and the absolute dissipation rates inferred from them for different measurement methods. The raw data for Figs. 7:1-7:4 is the same as is used in chapter 6 to present the spectra by HWA and phase shift. Fig. 7:5 is the result from PIV in counter rotation flow.
Fig. 7:1. (a) Third order moment by HWA method (from time domain) at h=-30mm. (b) Fitted dissipation rate according to the 4/5 law.

Fig. 7:2. (a) Third order moment by HWA method at h=0. (b) Fitted dissipation rate according to the 4/5 law.
Fig. 7:3.  (a) Third order moment by phase shift method (from time domain) at h=−30mm.  (b) Fitted dissipation rate according to the 4/5 law.  Approximately 10 points in each Reynolds number.

Fig. 7:4.  (a) Third order moment by phase shift method (from time domain) at h=0.  (b) Fitted dissipation rate according to the 4/5 law.  Approximately 10 points in each Reynolds number.
The PIV analysis is based on 300 pairs of frames at each rotation speed. The pairs of frames were acquired at a rate of 2Hz with separation of 0.5 or 1.0ms. The moments were derived by correlating the velocity components in the y direction (the axis not distorted by the tilting of the camera) along the y axis (longitudinal correlation). The averaging was performed on the available lines in the velocity maps, totally yielding $4 \cdot 10^5$ points. Thus we obtain true spatial information of velocity difference moments.

The presented plots of third order moment over separation distance and over dissipation rate are scaled to non-dimensional form such that the peak is at 4/5. The dissipation rates are fitted from this scaling. The plots in Figs.7:3a,7:4a are produced based on average of 10 sets of measurements of 18 seconds acquisition time. The small scatter of the 10 points at each Re in Figs.7:3b,7:4b demonstrates the repeatability of the dissipation rates. In comparison between the methods the values of dissipation rate are not always as expected (the reference line show the estimated values according to Eq.(3.2)), but the differences seem arbitrary and minor. In general, we obtain a power law of 3 in the Reynolds (except for HWA results in the shifted plane where the exponent is 4.4±0.1). Based on the generally comparable values of the peak heights of the third order moment from PIV analysis (spatial domain), (b) Fitted dissipation rate.
moments we can conclude that Eq.(7.6), in the summary of our hypothesis, is supported by the experiment.

Only in some range of separation distances the $S_{3/x}$ plots appear similar between the methods, taking into account that the $x$ scale in the phase shift plots remains uncertain if the Taylor conversion factor was overestimated (by conclusion of sec. 6.3). At low separation distances the exploding tendency of the phase shift $S_{3/x}$ plots is perhaps an artifact due to short time correlations of noise or acoustic reverberation. As for the position of the peak the situation varies between methods. The pumping length scale according to the HWA analysis (Figs.7:1-7:2) fitted by Eq.(7.9) is found: $L_f = 24 \pm 1 \text{mm}$ and $L_f = 23 \pm 1 \text{mm}$ (for $h=-30\text{mm}$ and $h=0$ cases respectively). However, there is no obvious decay over the distance in the plots by the phase shift and PIV methods, and the pumping length scale does not appear. We should remember that the sampling methods involve spatial integration: in the case of PIV the laser beam width is 3mm and a velocity vector is based on approximately 1x1mm interrogation window; in the case of phase shift the width of the sound beam is 10mm; in the case of HWA the length of the active wire is 1.25mm. We cannot claim that HWA is uniquely sensitive to the relevant flow features of the pumping scale. Hence, the existence of a pumping term in the Karman-Howarth equation is still questionable.

7.4. PDF of HWA and sound

The probability distribution function (PDF) of velocity differences (or phase shift differences) is complimentary information to the spectrum and the variance of fluctuations. The PDF reveals one aspect of turbulence, the one that involves the cascade mechanism or, specifically, the fluctuations in the energy transfer function. A convenient way to summaries the results is by fitting them with a model described by [Castaing & al 1990]. The model was used with success also by [Renner 2001]. The idea is to build an ensemble of Gaussian distributions with a varying width according to a log-normal distribution. Accordingly, the probability of a velocity difference $\delta v = v_x(x'+x) - v_x(x')$ is estimated as:
\[
\text{PDF}(\delta v) = \int_0^\infty \frac{\sigma}{2\sigma^2} \exp(-\ln^2(\sigma / \sigma_0)) C(\frac{\delta v}{\sigma}),
\]
(7.10)

where \(\sigma_0 \equiv <(\delta v)^2>^{1/2}\), and \(C(\frac{\delta v}{\sigma})\) is the skewness function approximated as:
\[
C(\frac{\delta v}{\sigma}) = A(1 + a_s \frac{\delta v / \sigma}{[1 + (\delta v / \sigma)^2]^{1/2}}),
\]
(7.11)

where \(A\) is a normalization factor and \(a_s\) is the skewness parameter that was determined from HWA data (by Castaing) as \(a_s = 0.18\). Additionally one needs to shift \(\delta v\) so \(<\delta v > = 0\). The explanation of the skewness, according to Castaing, is that the distribution of \(\delta v(x)\) does not depend directly on \(x\) but on the separation distance \(\tilde{x} = x - v_x \tau_c\) that is remembered from a time \(\tau_c\) before measurement (\(\tau_c\) concerns with vortex stretching relaxation time). If we assume the skewness parameter is fixed we are only required to find the fitting values of \(\Lambda\) in Eq.(7.10) and then find an exponent \(\beta\) according to
\[
\Lambda^2(x) \sim x^{-\beta}.
\]
(7.12)

We can check if \(\beta\) has some relation to the level of intermittency, and whether the same values are obtained from HWA and sound phase shift data in counter rotation flow.

In Fig.7:6a the presented PDF plot of HWA data looks familiar to workers in turbulence: For short separations it has an exponential tail and for long separation it is Gaussian (as plotted in half log scales). The values of \(\Lambda\) from the fits are relatively large compared with the values from the phase shift method (Fig.7:6b), for the flow of the same rotation speed and plane (180rpm, h=-30mm). Hence, the phase shift PDF is more closely a Gaussian than the HWA result. For the HWA PDF the fits are plugged with the skewness parameter \(a_s = 0.18\) and the plot is indeed not symmetric for positive and negative values of \(\delta v\). For the phase shift PDF the skewness parameter is zero. The model seams to partly demonstrate both measurements in the same category.

In Fig.7:7 the values of \(\beta\) derived based on Eq.(7.12) from a slope of the PDF fits are plotted for the HWA data. A consistent dependence appears, resembling somehow another intermittency indicator shown in Fig.7:8b (discussed in the next section). For the phase shift data no robust plot of \(\beta\) could be drawn. In summary, the PDF of the sound phase shift and HWA data are not comparable.
Fig. 7.6. (a) PDF of velocity difference by HWA. (b) PDF of velocity difference extracted from phase shift fluctuation in sound measurements. Both for counter rotation at 180rpm (Re=740,000) at the plane $h=-30\text{mm}$.
7.5. Extended Self Similarity (ESS)

The narrow inertial range, the one where the condition $-S_3/x = \frac{4}{5} \varepsilon$ is satisfied, presents a difficulty to obtain the exact power law scaling between $S_n$ of various order $n$'s. We follow the approach of [Benzi & al 1993] and define:

$$Z_n(x) = \langle |v_x(x') + x - v_x(x')|^n \rangle$$

(7.13)

The introduction of absolute differences of velocity allows to extend the range where the power law scaling

$$Z_n - (Z_3)^{\zeta(n)}$$

(7.14)

appears. $\zeta(n)$ are the ESS exponents that have the same meaning of the [Kolmogorov 1962] exponents. For small values of $n$ (e.g. $n<10$) we can fit a single value $I_\zeta$ that concludes the exponent results:

$$\zeta(n) = \left(1 + I_\zeta^2\right)n - \frac{1}{3} I_\zeta^2 n^2$$

(7.15)
based on a fixed value \( \zeta(3) = 1 \) and a quadratic deviation from the [Kolmogorov 1942] value \( \zeta(n) = n/3 \). \( I_\zeta \) is interpreted as the intermittency level of a turbulent flow.

The results are plotted using error bars that are calculated from the fit as:

\[
\text{err}(I_\zeta) = 0.5I_\zeta < \left| \frac{\zeta(n)_{\text{fitted}} - \zeta(n)_{\text{exp}}}{\zeta(n)_{\text{exp}}} \right|_{\text{15 \%}}^{16}
\]  

(7.16)

Fig. 7:8. Intermittency parameter from scaling in velocity difference moments fitted by Eq.(7.15). (Left) Phase shift measurements, an average on 10 samples per each rotation speed. (Right) HWA measurements. The values obtained by the phase shift method are much lower than by HWA. (Both measurements at the plane \( h=-30\text{mm} \)).

In Fig.7:8 we show that the indicators of intermittency \( I_\zeta \) extracted from the sound phase shift measurements are much smaller than the values obtained by HWA. One possible explanation may hang on the width of the sound beam (of 10mm). The sensor integration along the vertical axis results in effective canceling of the 3D special features (such as worms) that generate the high levels of intermittency in HWA measurements. The conclusion is the same as derived from the PDF data: The phase shift fluctuations and the local velocities have different intermittency levels, yet they possess the same form, such as the power law scaling between the various order structure functions.
Chapter 8

Conclusions

Over this work we discovered that the use of multi-channel sound acquisition provides the following opportunities:

1) It is feasible to construct the far field from analysis of the sampled acoustic wave on intermediate plane of propagation. This method provides snapshots of the Fourier structure functions of velocity and vorticity, via scattering analysis, which allows a direct observation of the structure of the flow.

2) The spatio-temporal map of absolute velocity difference $|\Delta v_p|$, measured across acoustic elements and time (from the sound phase shift), provides a possibility to estimate the advection velocity to be used in the Taylor conversion between time and space in a turbulent flow.

3) The availability of many channels provides a fast statistical convergence.

We derived from the various measurement techniques the results of kinetic energy spectrum in counter rotation flow. The scattering technique (that involves the far field construction) is mainly approved by revealing exactly as expected the -5/3 power law of the spectrum, which is typical for the inertial range in turbulence. The spectrum is also checked by comparison to the hot-wire anemometry (HWA) spectrum, taking into account that a certain mismatch in magnitudes of the spectra is due to a weakness in the Taylor hypothesis. Kinetic energy spectrum based on phase shift fluctuations is another method which shows not quite acceptable results due to our experimental conditions. There is a limitation that at high sound frequencies the phase shifts at intense flow become ambiguous in $2\pi$, whereas at lower frequencies the geometric acoustics fails particularly in high $k$ details of the spectrum. Particle image velocimetry (PIV) was not arranged to provide significant spectrum information due to resolution limitation of the camera. Nevertheless, the kinetic energy spectra by either method exhibit remarkable scaling property by which the relative dissipation rates can be fitted and be presented versus the Reynolds number.
The level of velocity fluctuation by HWA was compared to Phase shift fluctuation measurements scaled in velocity units (interpreted as the average of velocity fluctuation over the acoustic path, denoted as \( v_p \)). Such comparison allowed estimating the integral scale of the flow, which was found \( L_1 = 51 \pm 4 \text{mm} \). The ratio of the integral scale over the length of acoustic path, \( 2L_1/L \), was found comparable to the square root of the ratio between the peaks in the plots of the third order moments of the phase shift and HWA, \( [S_{p3}/x]_{\text{peak}} /[S_3/x]_{\text{peak}} \), which allow to obtain the absolute dissipation rates solely from phase shift measurements (if \( L_1 \) is known). Fitting the HWA third order moment to the formula of [Moisy & al 1999] revealed that the suggested pumping scale is \( L_f = 23 \pm 1 \text{mm} \) in all the points visited by the probe (for comparison Moisy & al found \( L_f = 12 \pm 3 \text{mm} \) in a cell of 2/3 the size of ours). The value is reasonable in comparison to the integral scale that we found.

The Taylor hypothesis should be replaced by the Tennekes hypothesis in the case that the mean velocity is small compared to the mean fluctuation [Tennekes 1975]. There is a correlation between chaotic velocity difference in time and space, although the observed proportionality factor is neither fixed at the rms of velocity nor the average of the absolute velocity \( \langle |v| \rangle \) as tested here. Our experimental results of the spatio-temporal map of \( |\Delta v_p| \) is the clearest demonstration of the Tennekes hypothesis. The factor between time and space was determined around 2-3 times less than the average of absolute velocity (the advection velocity), and possibly the relation is not linear. The measurements were applied at two planes, one at the middle between the plates and one shifted by 15% of the overall distance between the plates. No significant difference between the conditions was observed, indicating that the case of vanishing average velocity is not critical to the spatio-temporal symmetry of velocity fluctuations.

Further investigation using the ultrasound tool can be suggested to perform on sensitive fluids, such as polymer solutions or Liquid Helium, due to the non-perturbative character of the measurement. Further promising improvement of the tool should involve increasing the number of elements and playing with scattering of multiple beams.
Appendix A

A.1. Relation between velocity and vorticity in the Fourier space

The following is based on assumed incompressibility of the flow and properties of Fourier transforms.

\[ F_{k_y} ((\nabla \times \vec{v})_z) = \frac{1}{(2\pi)^2} \int \exp(-ik_y \cdot \vec{r}')(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y})d^2r', \]

Gradients in the z direction are unimportant thus:

\[ \frac{\partial v_z}{\partial x} = -\frac{\partial v_y}{\partial y} \Rightarrow i\hat{y} \cdot \hat{k}_y F_{k_y} \{v_y\} = -i\hat{x} \cdot \hat{k}_y F_{k_y} \{v_x\}, \]

hence

\[ F_{k_y} \{\frac{\partial v_y}{\partial x}\} = F_{k_y} \{\hat{x} \cdot \nabla v_y\} = i\hat{x} \cdot \hat{k}_y F_{k_y} \{v_x\} = -i\frac{(\hat{x} \cdot \hat{k}_y)^2}{(\hat{y} \cdot \hat{k}_y)} F_{k_y} \{v_x\}. \]

and in the same way

\[ F_{k_z} \{\frac{\partial v_z}{\partial y}\} = F_{k_z} \{\hat{y} \cdot \nabla v_z\} = i\hat{y} \cdot \hat{k}_z F_{k_z} \{v_x\}. \]

Finally we obtain the linear relation

\[ F_{k_z} ((\nabla \times \vec{v})_z) = -i\frac{(\hat{x} \cdot \hat{k}_z)^2 + (\hat{y} \cdot \hat{k}_z)^2}{(\hat{y} \cdot \hat{k}_z)} F_{k_z} \{v_x\} = -i\frac{k_z^2}{(\hat{y} \cdot \hat{k}_z)} F_{k_z} \{v_x\}. \]

(9.1)

A.2. Energy spectrum in the case of isotropic turbulence

By assuming homogeneity and isotropy the following kinetic energy per unit mass is contained in the 3D flow:

\[ E_r = \frac{3}{2A} \int d^2r' v^2(r'), \]

where A is the sampled area. Due to Parseval relation:
\[ \int d^2 r' \nabla'^2 = (2\pi)^2 \int d^2 k |F_k \{v_s\}|^2 \]

Using Eq.(9.1) we arrive at:

\[ E_r = \frac{3(2\pi)^2}{2A} \int d^2 k \left( \frac{\hat{v} \cdot \hat{k}}{k^4} \right)^2 |F_k \{(\nabla \times \bar{v})_s\}|^2 \]  

(9.2)

The energy spectrum \(E(k)\) is defined by \(E_r = \int dk E(k)\), and by replacing in Eq.(9.2) \(d^2 k = k \cdot dk \cdot d\theta\) we finally obtain:

\[ E(k) = \frac{3(2\pi)^2}{2A} \frac{1}{k} |F_k \{(\nabla \times \bar{v})_s\}|^2 \int d\theta \cos^2 \theta = \frac{6\pi^3}{Ak} |F_k \{(\nabla \times \bar{v})_s\}|^2 \]  

(9.3)

Appendix B

B.1. Fourier transform in the case of axisymmetry

In the following derivation we apply the axisymmetry of a single vortex and represent the velocity field by the azimuthal velocity profile, \(v_t\), and use properties of the Bessel functions \(J_0(x) = -J_1(x)\).

\[ F_{k_s} \{v_s(r')\} = F_{k_s} \{v_t(r')\} \hat{\gamma} \cdot \hat{k}_s \frac{\partial}{\partial k_s} \frac{v_t(r')}{r'} = \left( \frac{\hat{\gamma} \cdot \hat{k}_s}{k_s} \right) \frac{i\partial}{\partial k_s} F_{k_s} \{v_t(r')\} \]

\[ = \frac{1}{(2\pi)^2} \left( \frac{\hat{\gamma} \cdot \hat{k}_s}{k_s} \right) \frac{i\partial}{\partial k_s} \int d^2 r' \exp(-i\hat{k}_s \cdot \hat{r}') \frac{v_t(r')}{r'} \]

\[ = \frac{1}{(2\pi)^2} \left( \frac{\hat{\gamma} \cdot \hat{k}_s}{k_s} \right) \frac{i\partial}{\partial k_s} \int dr' 2\pi r' J_0(k_s r') \frac{v_t(r')}{r'} \]

\[ = \frac{-i(\hat{\gamma} \cdot \hat{k}_s)}{2\pi k_s} \int dr' v_t(r') r' J_1(k_s r') \]
B.2. Phase shift slope with a point-like emitter and a single vortex

The idea of deriving the slope of phase shift at the forward direction is as follows. Since we check the acoustic paths near the forward direction, and the vicinity of the center of the single vortex contributes negligible part of the phase shift, then we regard the center of the vortex the same as the centers of the paths, when concerning the velocity field in the path coordinates. Let us define the transformation from the usual frame \((x', y')\) to the coordinates of a path \((s', u')\) that opens at direction \(\theta/2\) from the forward direction (see figure below) in a cell of radius \(r_c\).

\[
x' = \cos \frac{\theta}{2} s' - \sin \frac{\theta}{2} u' - r_c \\
y' = \sin \frac{\theta}{2} s' + \cos \frac{\theta}{2} u'
\]

Define the spherical coordinates:

\[
x'/r' = \cos \theta'
\]
\[
y'/r' = \sin \theta'
\]
Thus the index of refraction concerned with the entrainment by velocity field is

\[
n^2(\vec{r}', \hat{s}) \approx 1 - \frac{2\xi \cdot \vec{v}}{c} \\
= 1 - \frac{2v_i(r') \sin(\theta' - \frac{\theta}{2})}{c} \\
= 1 - \frac{2v_i(r')}{c} [\sin \theta' \cos \frac{\theta}{2} - \cos \theta' \sin \frac{\theta}{2}] \\
= 1 - \frac{2v_i(r')}{c} \left[ \frac{y'}{r'} \cos \frac{\theta}{2} - \frac{x'}{r'} \sin \frac{\theta}{2} \right] \\
= 1 + \frac{2v_i(r') r'}{c} \sin \frac{\theta}{2}
\]
Thus the phase shift is calculated

\[
\phi(\theta) = k_0 \int (n(\vec{r}', \hat{s}) - 1) ds' = \frac{k_0 r_c}{c} \sin \left( \frac{\theta}{2} \right) \int_0^{2\pi} \frac{v_i(s')}{r'(s')} ds' \\
= \frac{2k_0 r_c}{c} \sin \left( \frac{\theta}{2} \right) \int_0^{2\pi} \frac{v_i(r')}{r'} dr'
\]
Hence, by simple derivation Eq.(2.16) is obtained.
Appendix C

C.1. Definitions of the scales and numbers in turbulence

From dimensional consideration any scale of a flow (below the speed of sound) must originate from boundary interaction. A boundary at moving frame relative to the fluid exerts an external force, also known as the pumping force, which leads to some energy flux. In 3D flow the flux of energy is transferred to lower scales by a cascade mechanism, until it dissipates by viscous friction. Thus, in stationary conditions the energy flux is indicated by the rate of energy dissipation per unit mass, $\varepsilon$. The smallest scale, where motion is ceased, is called the Kolmogorov scale, $\eta$, calculated as

$$\eta = \left( \frac{\nu^3}{\varepsilon} \right)^{1/4} \quad (9.4)$$

where $\nu$ is the kinematic viscosity (for water $\nu = 0.01 \text{ cm}^2\text{s}^{-1} = 10^{-6} \text{ m}^2\text{s}^{-1}$).

The dissipation rate depends on a dimensionless control parameter known as the Reynolds number, $Re$, which we define here

$$Re = \frac{2\Omega R^2}{\nu} \quad (9.5)$$

where $\Omega$ is the rotation frequency of the plates (in rad/s) and $R$ is the radius of the plates; hence the denominator is a product of length and speed of the largest eddy.

The Taylor scale, $\lambda_T$, describes the ratio between the common velocity and the common gradient of velocity. The definition varies; according to [Piquet 1999; Mestayer 1982] the Taylor scale can be calculated from the second derivative of the longitudinal correlation function (defined in Eq.(6.7) ) as:

$$\lambda_T = \left( \frac{-2}{f''(0)} \right)^{1/2} \quad (9.6)$$

Experimentally the Taylor scale is often determined from the second moment of velocity difference:

$$\hat{\lambda}_T = \left[ \frac{< (v - \bar{v})^2 >}{< \delta v(x)^2 >} \right]^{1/2} \quad (9.7)$$
It is also possible to read the Taylor scale from the energy spectrum [Beran 1968 after Kraichnan]:

\[
\lambda_T = \left[5 \int_0^\infty dkE(k) / \int_0^\infty dkE(k)k^2 \right]^{1/2}
\]  (9.8)

Another idea to estimate the Taylor scale when experimental resolution is not sufficient is via the relation for isotropic turbulence \( \varepsilon = 15\nu |\nabla v|^2 \). Thus it is estimated:

\[
\lambda_T = \left[ \frac{15\nu \varepsilon_{\text{rms}}^2}{\varepsilon} \right]^{1/2}
\]  (9.9)

where \( \varepsilon_{\text{rms}} = \langle (v - \overline{v})^2 \rangle \).

There is common use in a dimensionless number \( R_\lambda \) defined as

\[
R_\lambda = \frac{\varepsilon_{\text{rms}}}{\nu},
\]  (9.10)

which is a good indicator for the level of turbulence.

The integral scale \( L_1 \) indicates the size of the largest eddy based on the normalized correlation function \( f(x) \) (following Eq.(6.8)):

\[
L_1 = \int_0^\infty dx f(x)
\]  (9.11)

The integral scale also can be read from the energy spectrum [Piquet 1999]

\[
L_1 = \frac{3\pi}{4} \int_0^\infty dkE(k)k^{-1} / \int_0^\infty dkE(k)
\]  (9.12)

\( L_1 \) is also known as the thickness of the boundary layer near a wall or in a wake.

In order to complete this survey we should mention the scale of mixing length defined locally as:

\[
l_m = \frac{\varepsilon_{\text{rms}}}{|\partial v/\partial s|}
\]  (9.13)

where \( \partial v/\partial s \) is the shear rate of the mean flow. Near the boundary at a distance larger than 0.2\( L_1 \) from the wall usually \( l_m \) is constant, and it is claimed empirically that

\[
l_m = 0.085 \cdot L_1 \) (the proportionality is universal) [Biswas & al 2002], so the two scales are in fact the same.

97
Appendix D

D.1. Analysis of Kraichnan’s source-term of sound

Choosing the $x_1$ axis along the incident wave direction the Kraichnan term [Kraichnan 1953] is handled as follows:

$$\frac{\partial^2 (v'_1 v_j)}{\partial x_i \partial x_j} = \frac{\partial v'_1}{\partial x_i} \nabla \cdot \vec{v} + \frac{\partial^2 v'_1}{\partial x_i^2} v_i + \frac{\partial^2 v'_i}{\partial x_i^2} \frac{\partial v_i}{\partial x_j} + \frac{\partial^2 v_j}{\partial x_i \partial x_j} v_i$$

(written using the summation rule).

The conditions are an incompressible flow, so $\nabla \cdot \vec{v} = 0$, and for sound $\partial v'_i / \partial x_j = i k \delta_{ij}$, and for high ultrasound frequencies one can assume $\partial^2 v_i / \partial x_i^2 \ll k^2 v_i$ (if the wavelength is smaller than the Taylor scale). Hence we arrive at a simplified term:

$$\frac{\partial^2 (v'_1 v_j)}{\partial x_i \partial x_j} = -k^2 v'_1 v_i + i k \frac{\partial v_i}{\partial x_i} = -k^2 \frac{\Psi}{c \rho_0} v_i + i k \frac{\Psi}{c \rho_0} \frac{\partial v_i}{\partial x_i}$$

(9.14)
Bibliography


**Publication**